High Performance Stencil Code Generation with Lift

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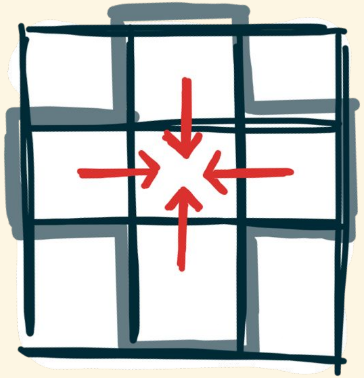
WWU MÜNSTER

THE UNIVERSITY of EDINBURGH

University of Glasgow

WHY STENCIL COMPUTATIONS?



Stencil computations are a class of kernels which update *neighboring* array elements according to a fixed pattern, called *stencil*.

Frequently occur in:

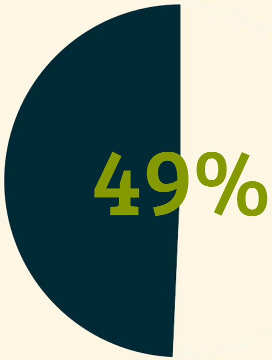
- Medical Imaging
- Physics Simulations
- Machine Learning
- PDE Solvers

WHY STENCIL COMPUTATIONS?

Stencil compute time:

HPC Center München

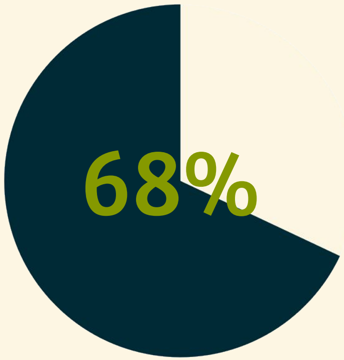
lrz



2017

HPC Center Stuttgart

HLRS



2016

Frequently occur in:

- Medical Imaging
- Physics Simulations
- Machine Learning
- PDE Solvers

YET ANOTHER STENCIL PAPER?

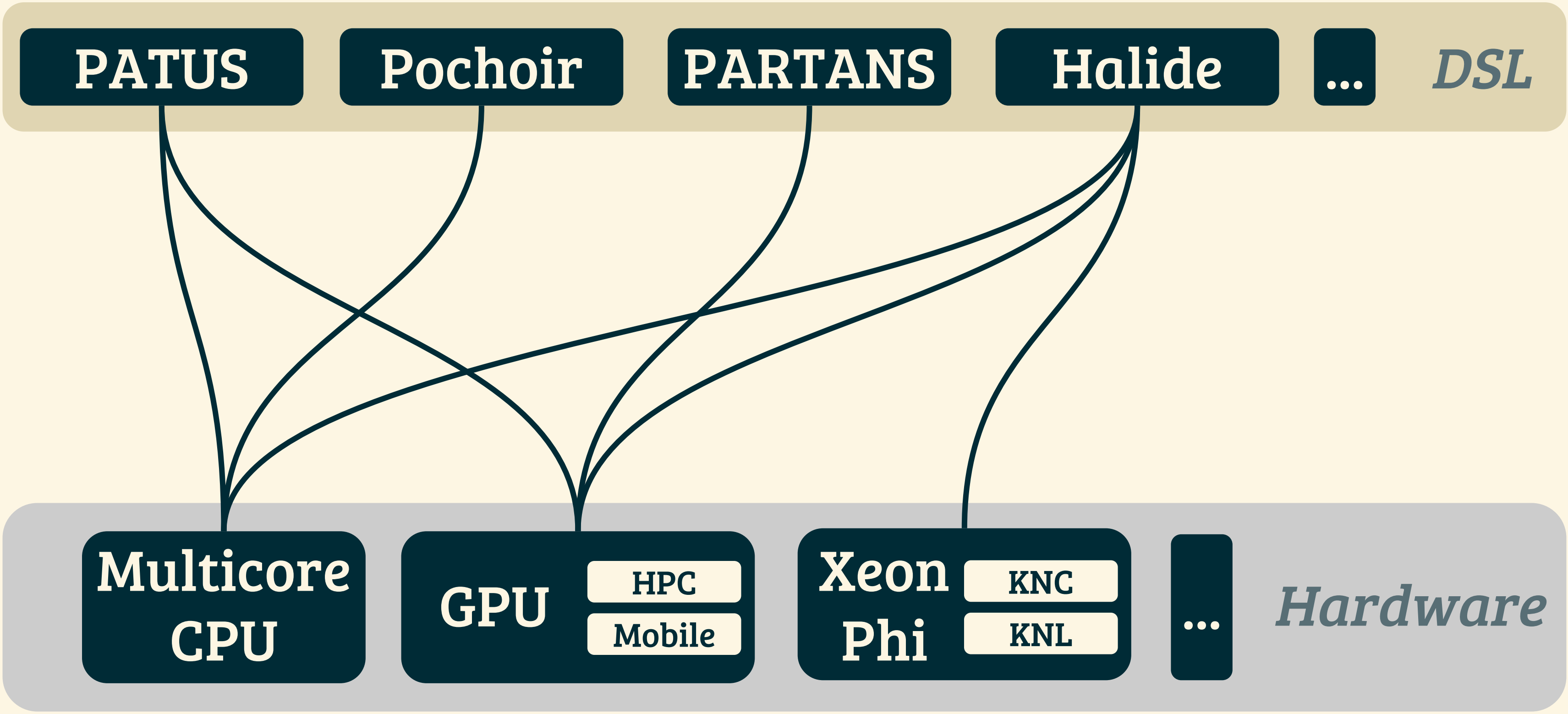
2005 2007 2009 2011 2013 2015 2018

ICS'05 PLDI'07 ICS'09 SC'10 CGO'12 CLUSTER'13 CGO'15 WOLFHPC'16 CLUSTER'17 CGO'18

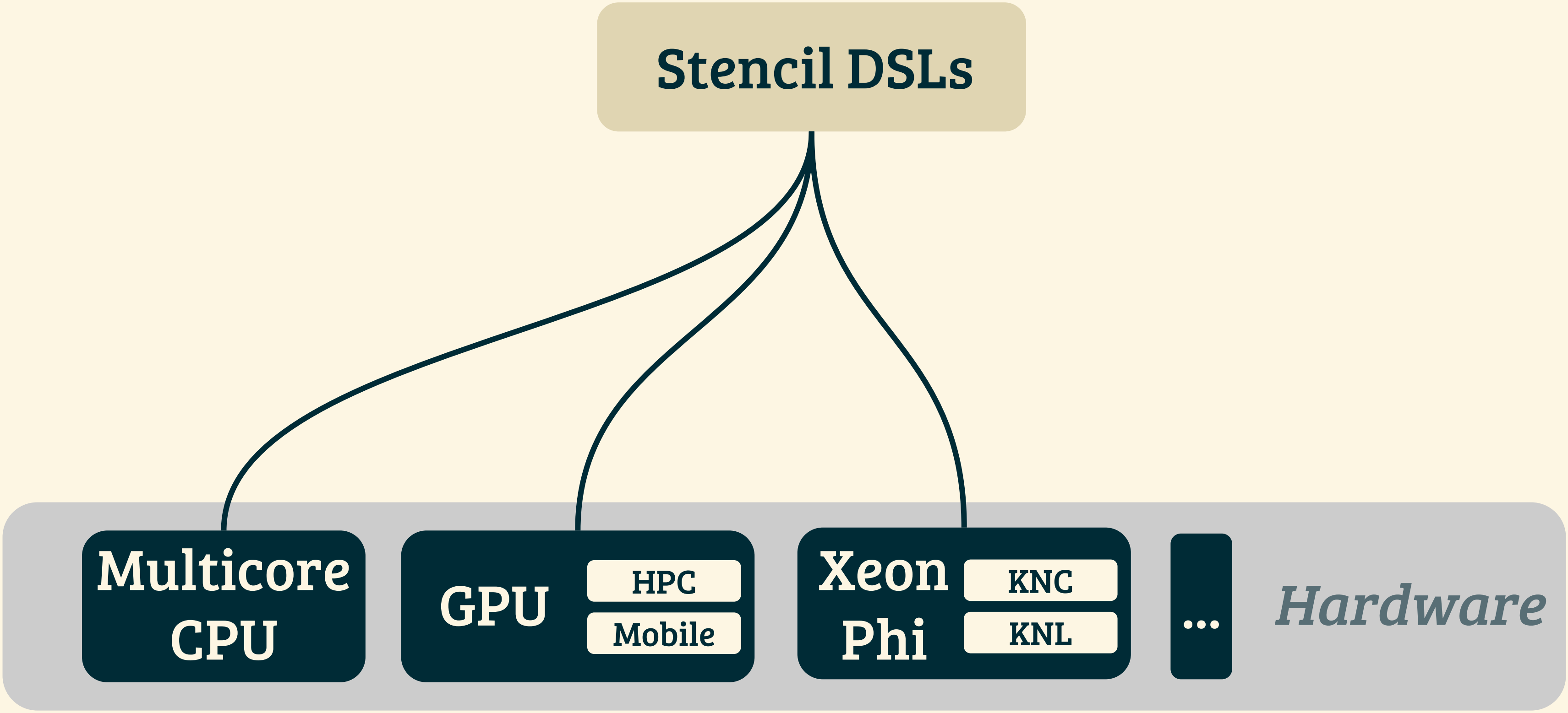
Domain Specific Languages

PATUS | Pochoir | PARTANS | Halide | ... | *DSL*

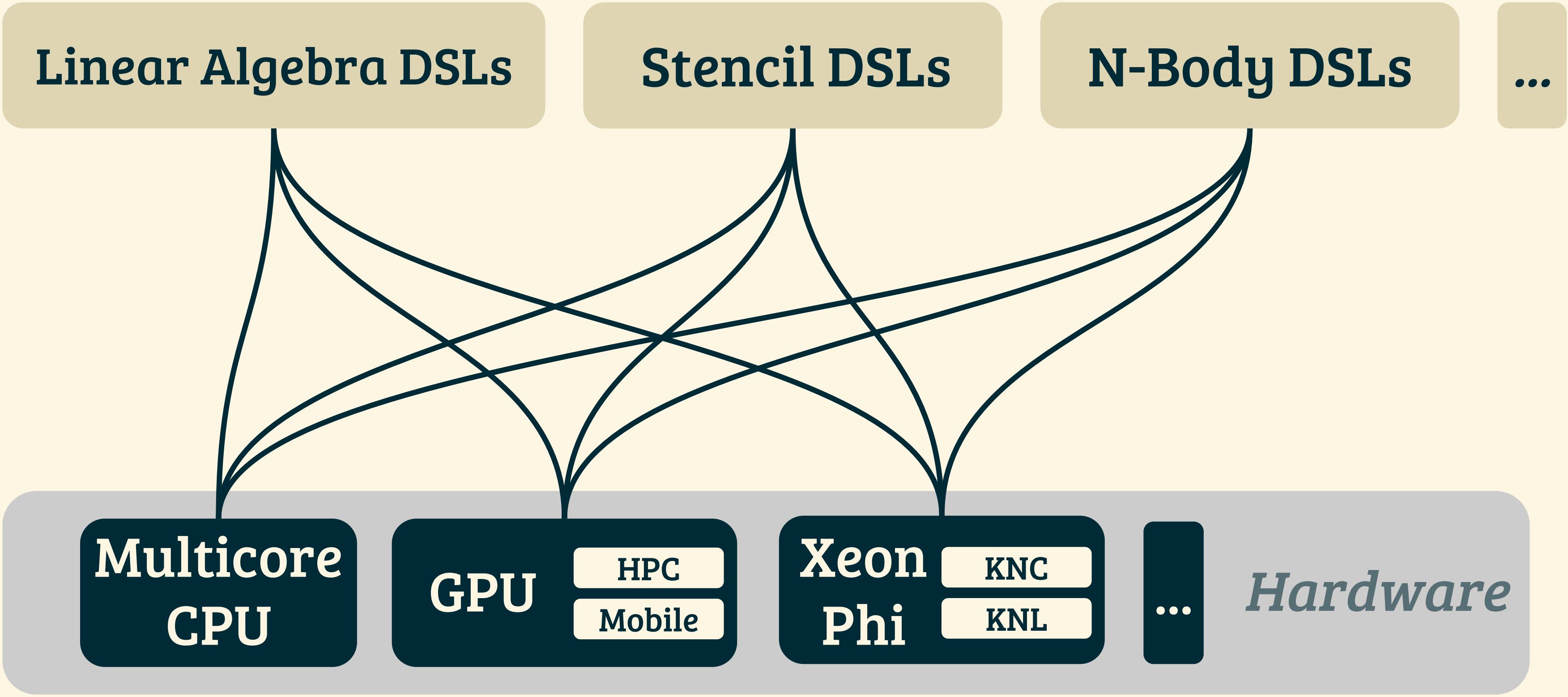
EXPLOITING DOMAIN KNOWLEDGE



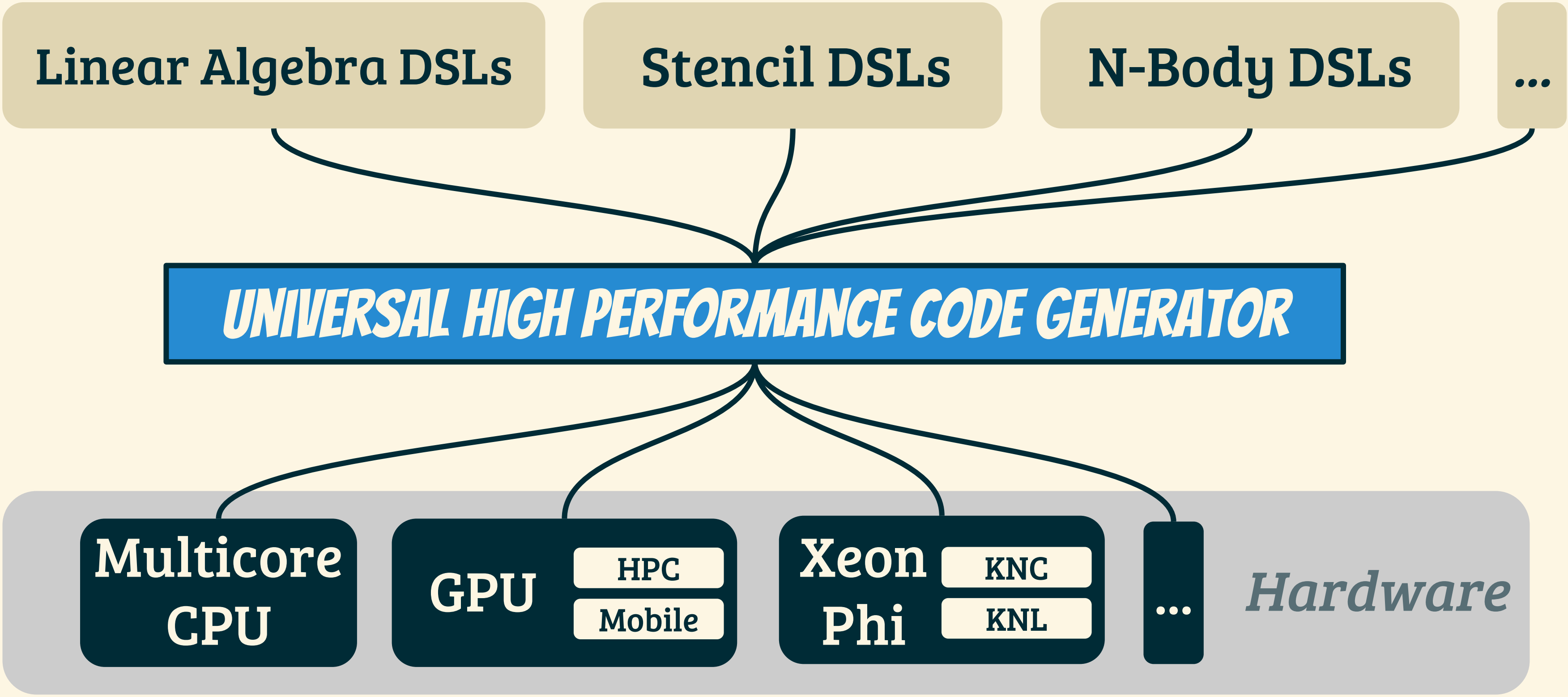
EXPLOITING DOMAIN KNOWLEDGE



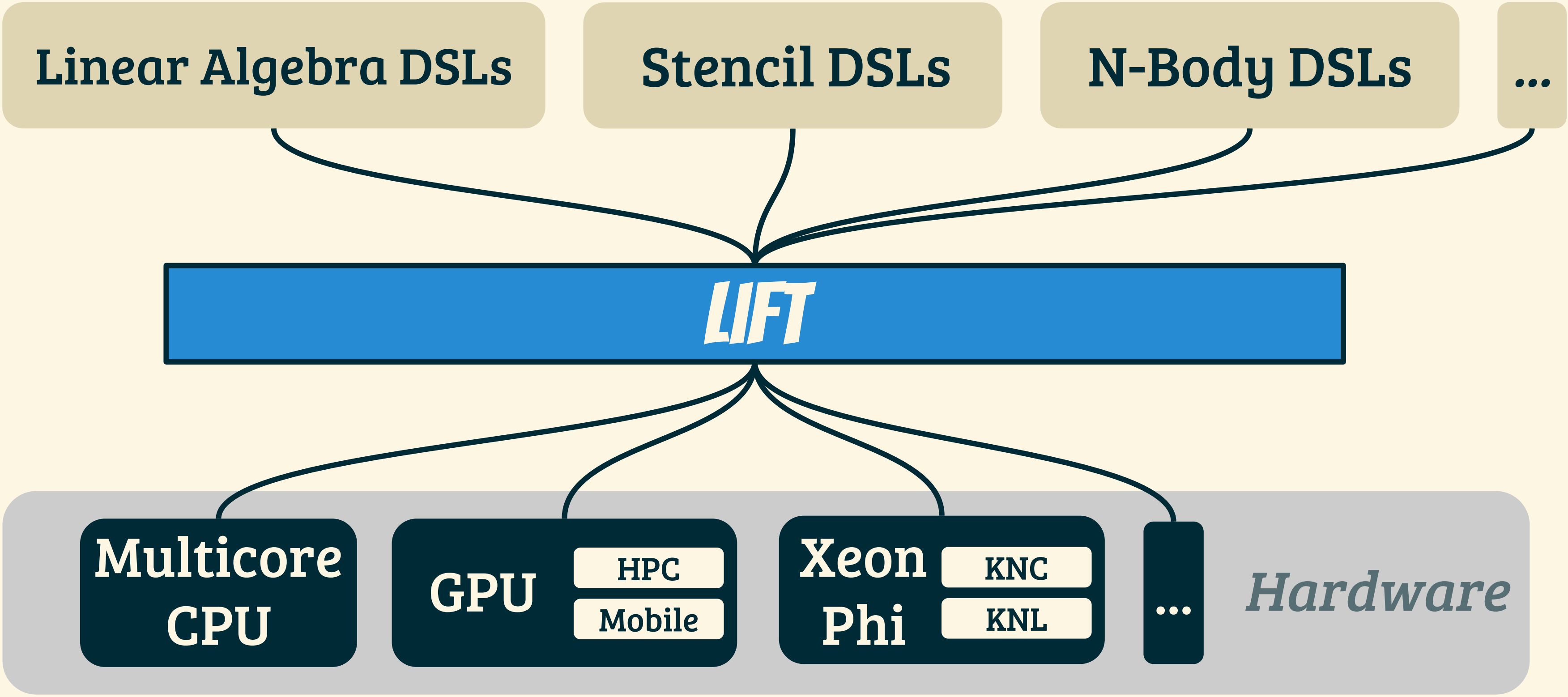
EXPLOITING DOMAIN KNOWLEDGE



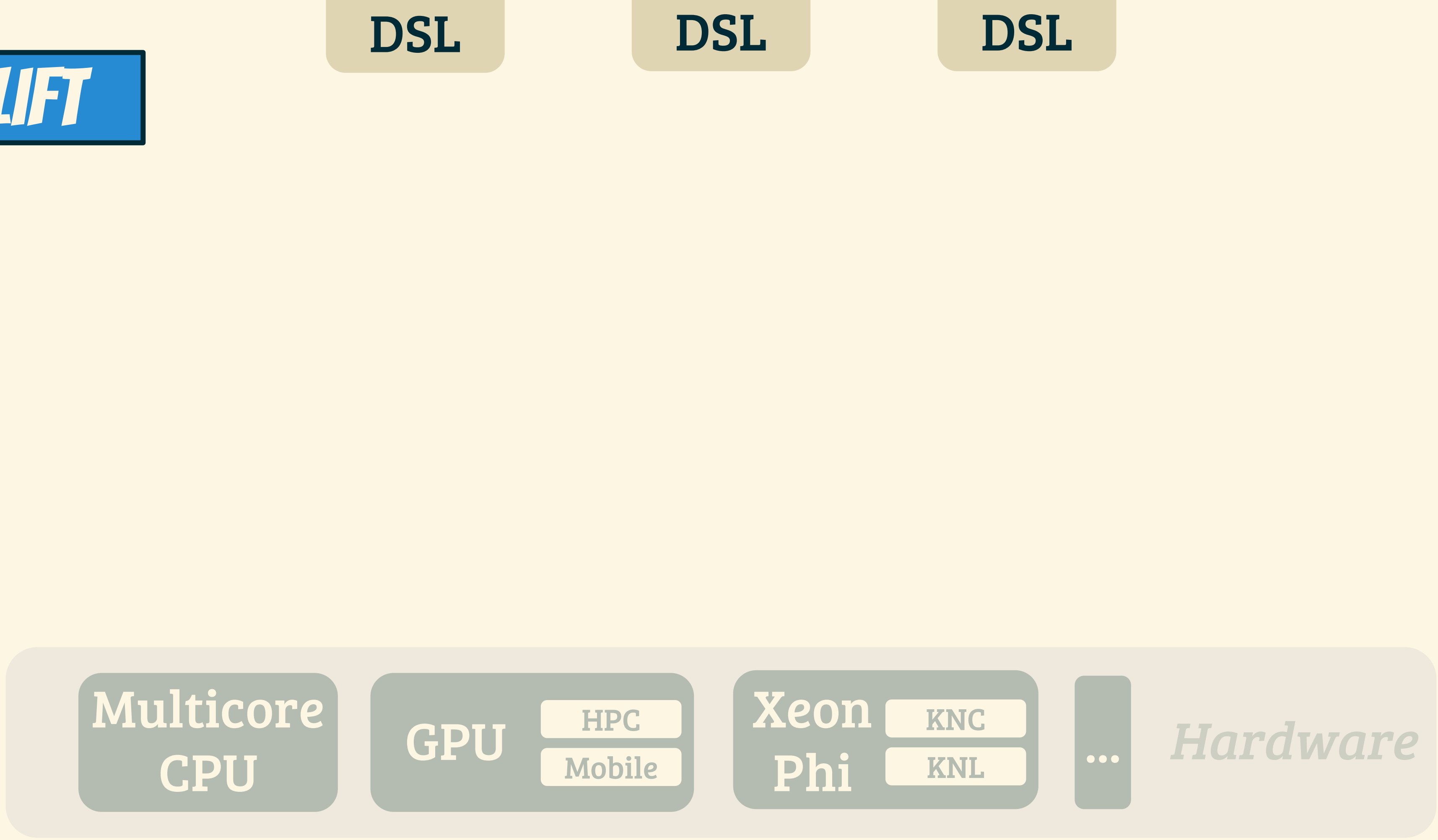
APPROACHING PERFORMANCE PORTABILITY



APPROACHING PERFORMANCE PORTABILITY



LIFT


DSL
DSL
DSL
Multicore CPU
GPU
HPC
Mobile
Xeon Phi
KNC
KNL
...
Hardware


DSL
DSL
DSL
High-Level IR
Multicore CPU
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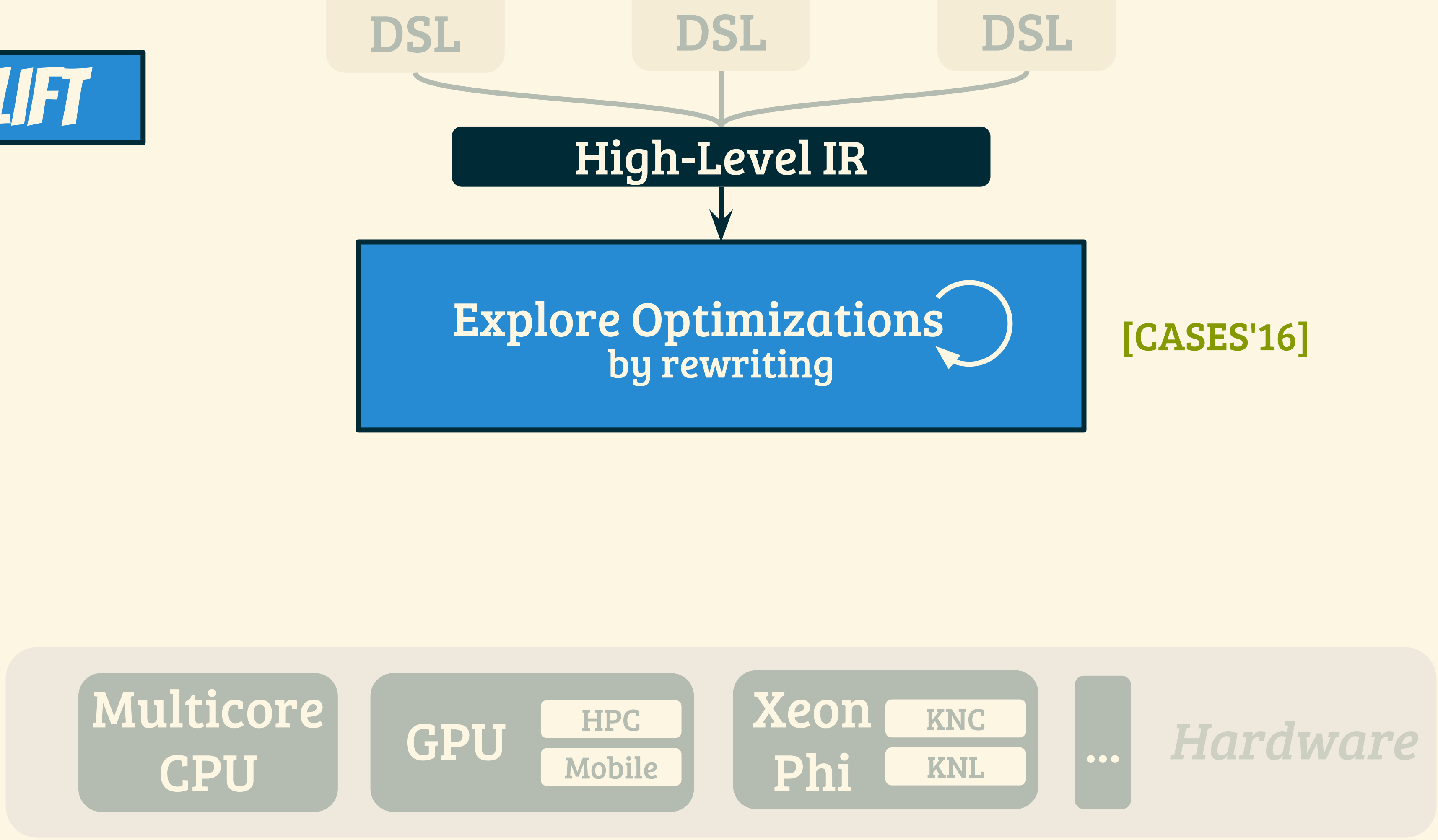
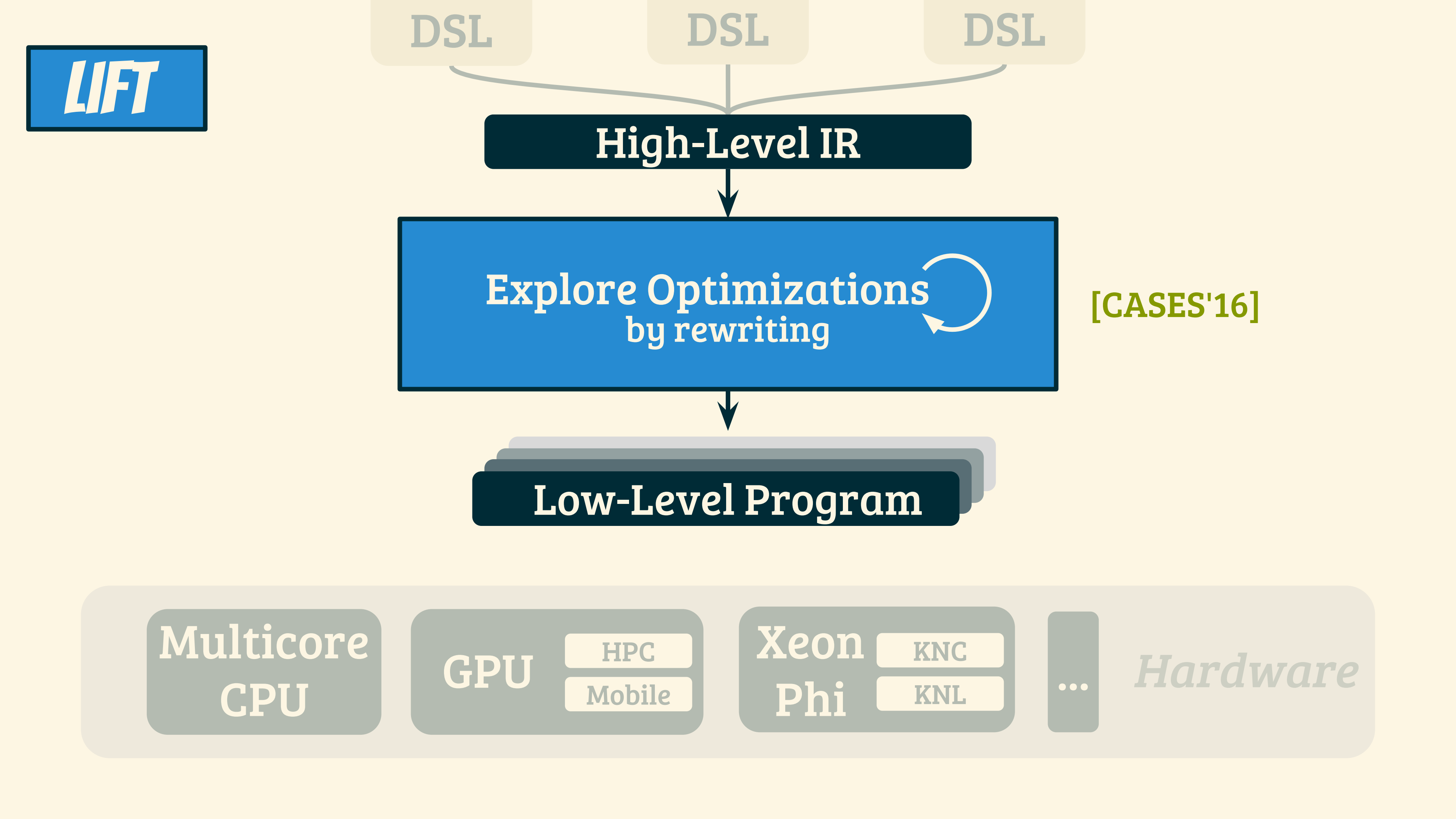
...
Hardware


DSL
DSL
DSL
High-Level IR
Explore Optimizations
by rewriting
Multicore
CPU
GPU
HPC
Mobile
Xeon
Phi
KNC
KNL
...
Hardware

[CASES'16]


LIFT
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High-Level IR
Explore Optimizations
by rewriting
[CASES'16]
Low-Level Program
Multicore CPU
GPU
HPC
Mobile
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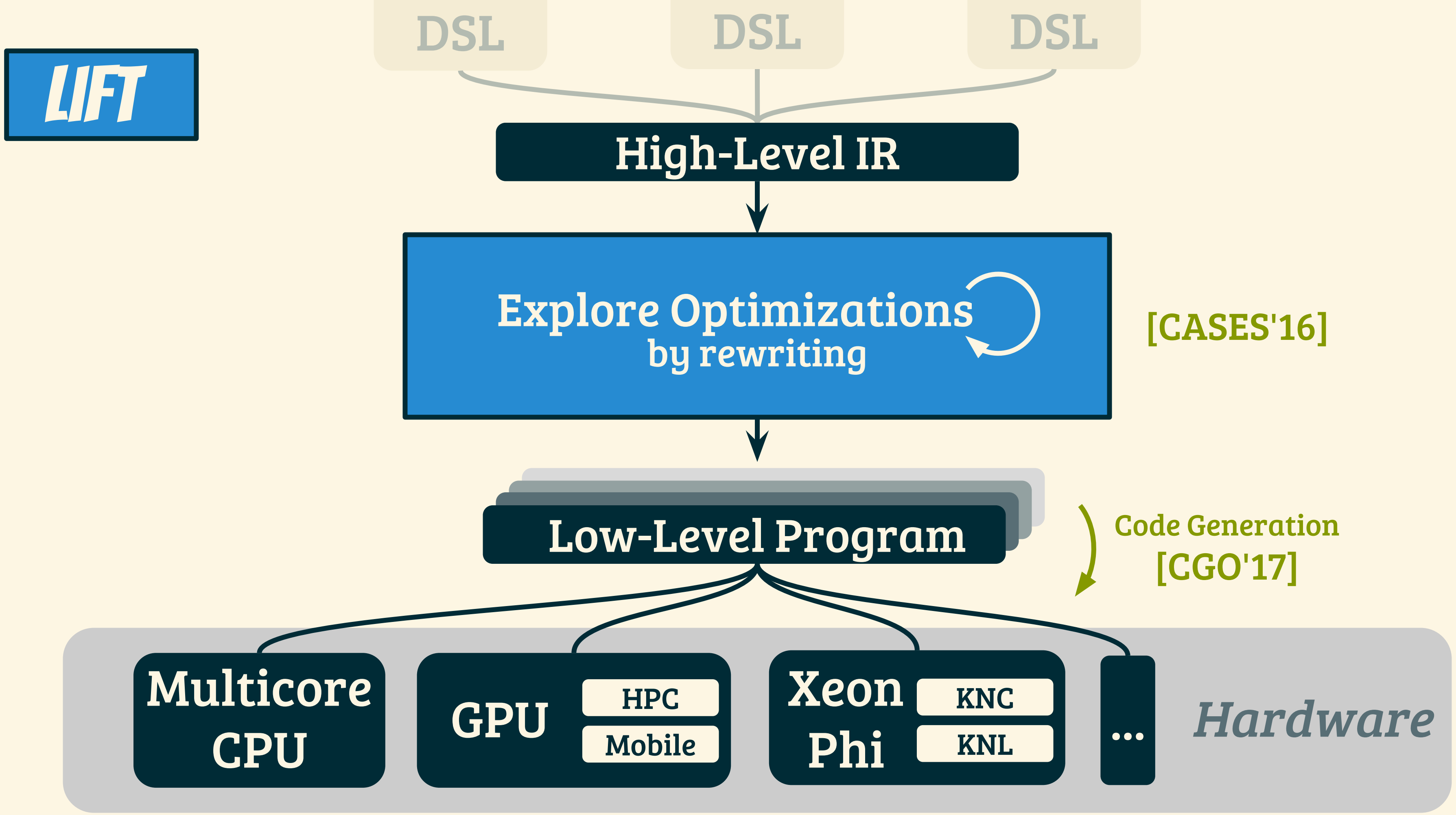
...
Hardware



LIFT
DSL
DSL
DSL
High-Level IR
Explore Optimizations
by rewriting
[CASES'16]
Low-Level Program
Code Generation
[CGO'17]
Multicore
CPU
GPU
HPC
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...
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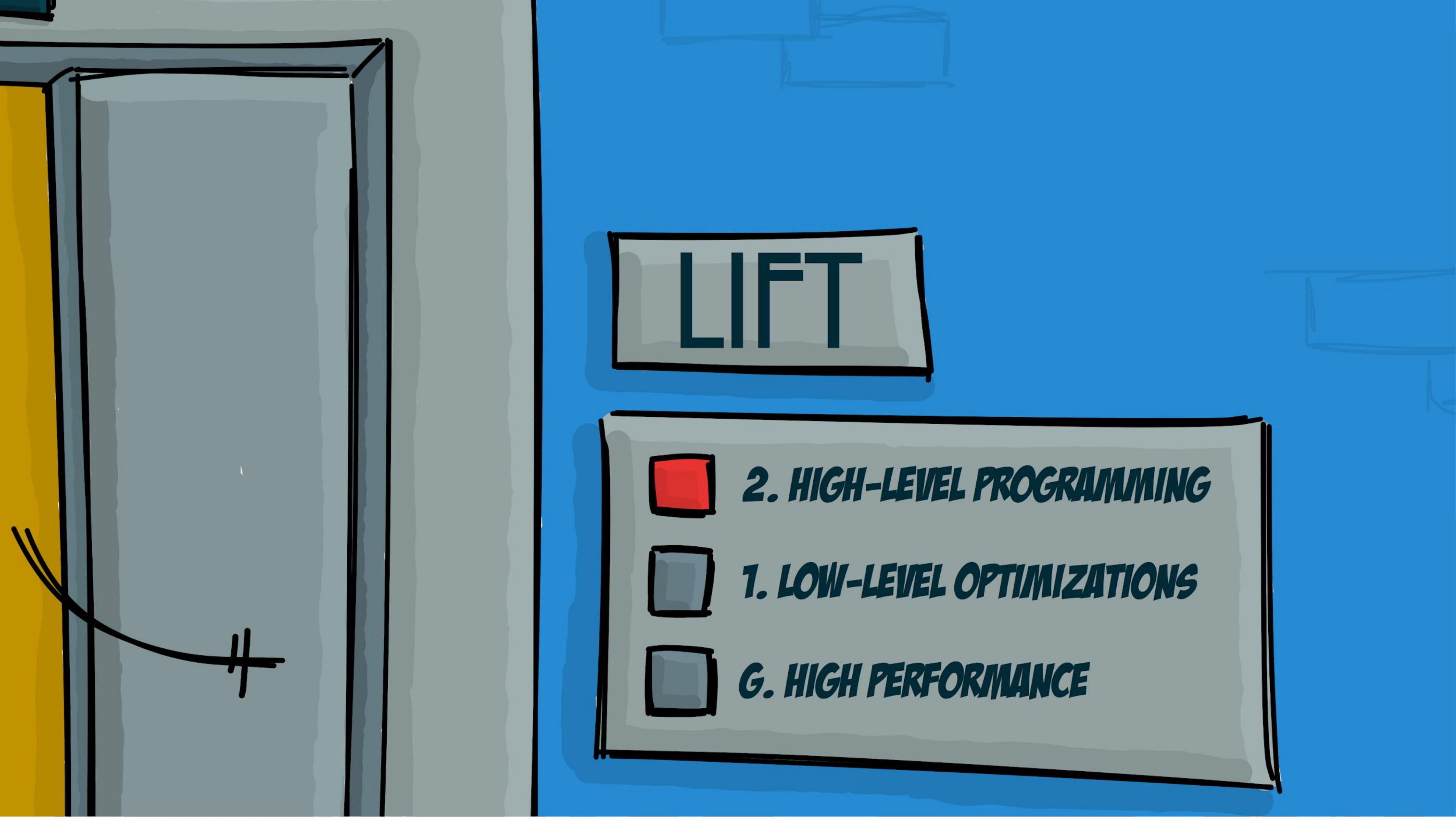
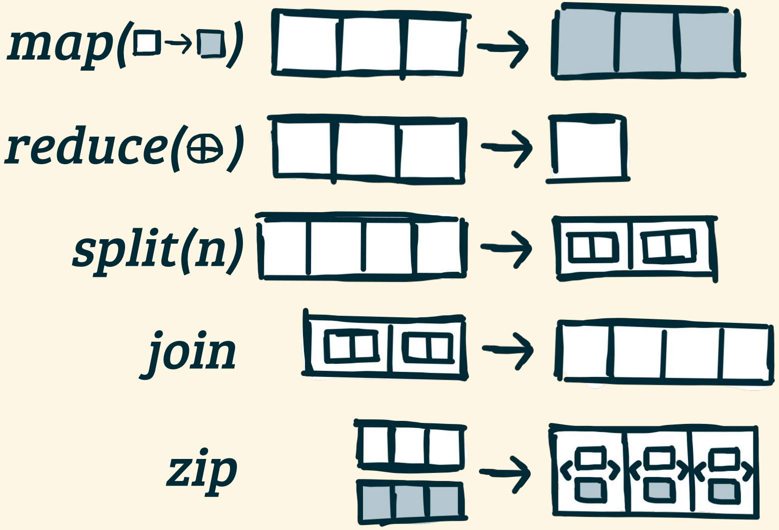

LIFT
2. HIGH-LEVEL PROGRAMMING
1. LOW-LEVEL OPTIMIZATIONS
G. HIGH PERFORMANCE

LIFT'S HIGH-LEVEL PRIMITIVES



LIFT'S HIGH-LEVEL PRIMITIVES

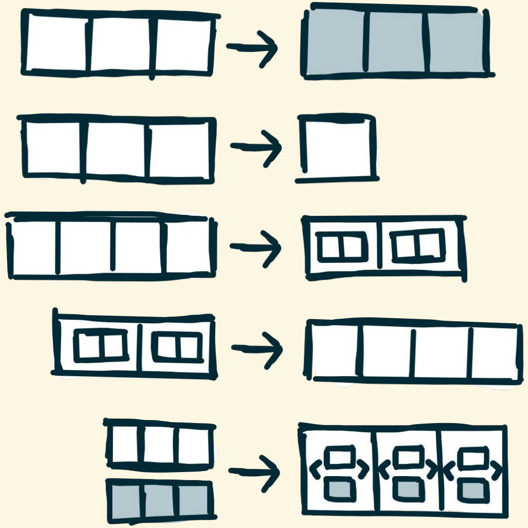
map(□→□)

reduce(⊕)

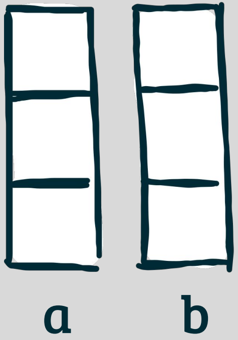
split(n)

join

zip



dotproduct.lift



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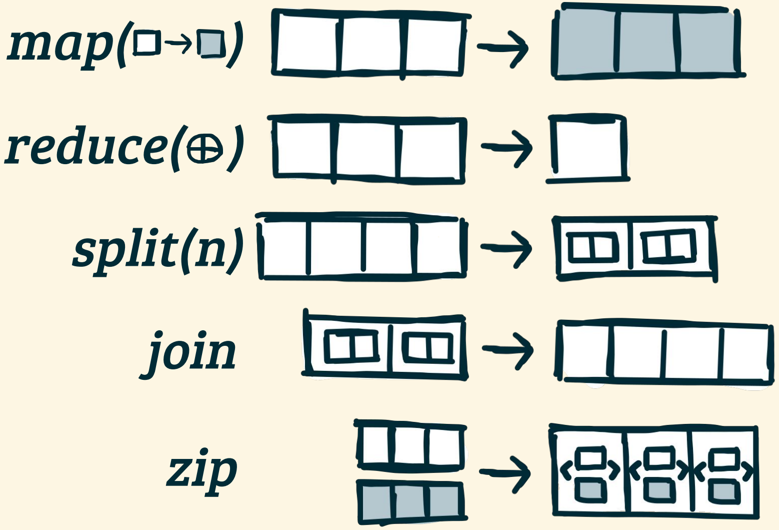

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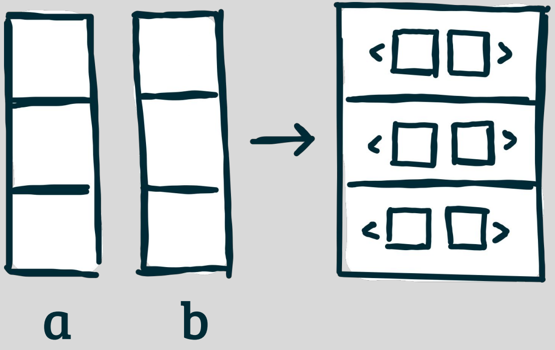
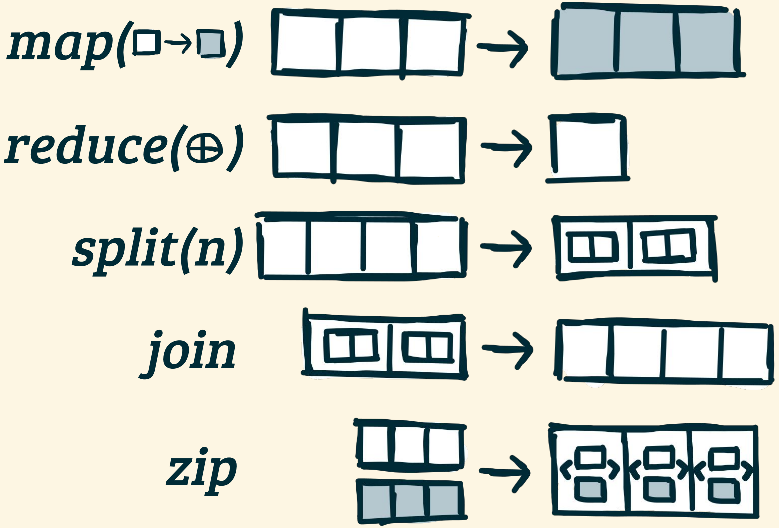
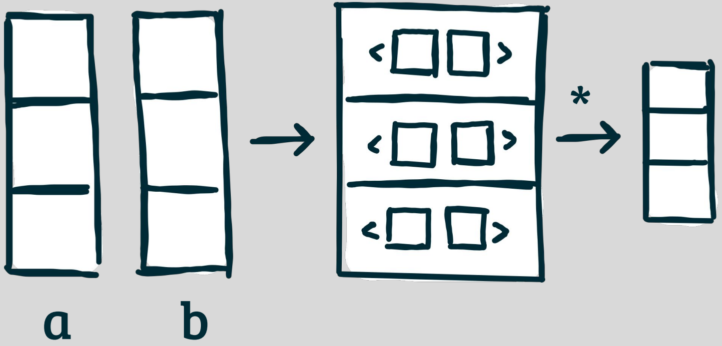



```
zip(a,b)
```

LIFT'S HIGH-LEVEL PRIMITIVES

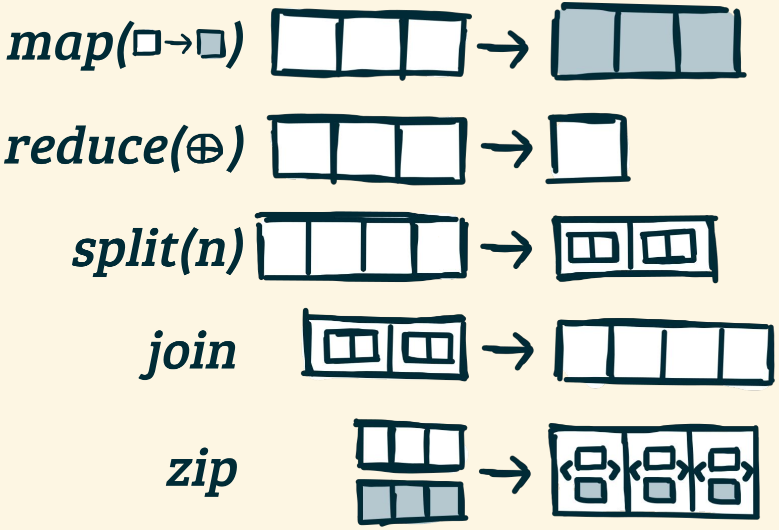


dotproduct.lift




```
map(*, zip(a,b))
```

LIFT'S HIGH-LEVEL PRIMITIVES



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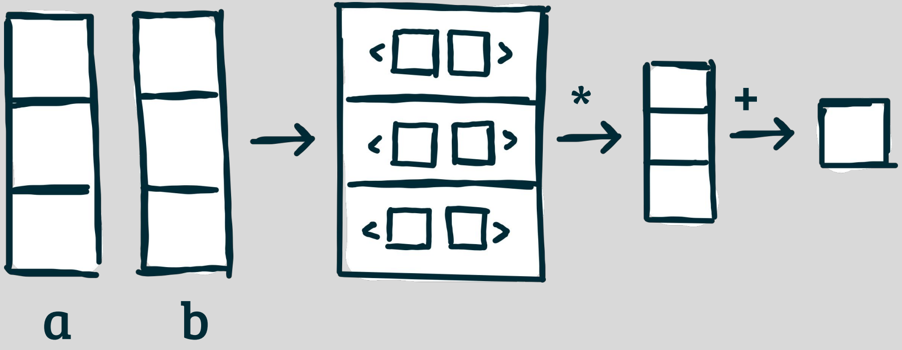
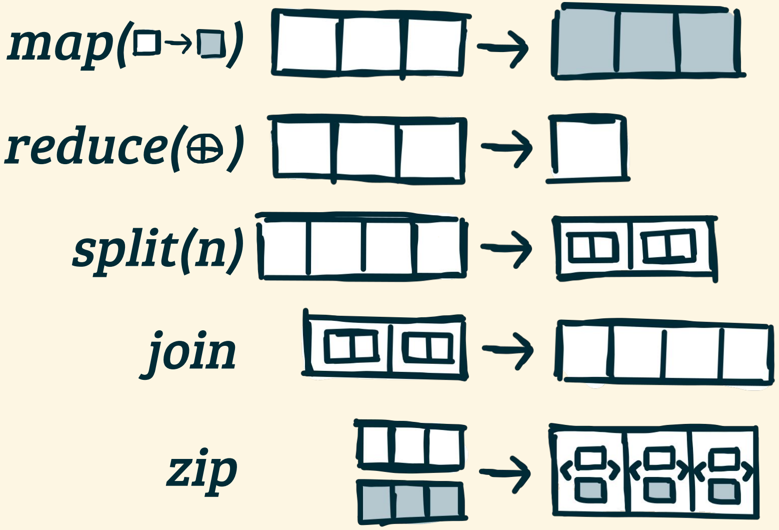



```
reduce(+,0, map(*, zip(a,b)))
```

LIFT'S HIGH-LEVEL PRIMITIVES



stencil.lift?

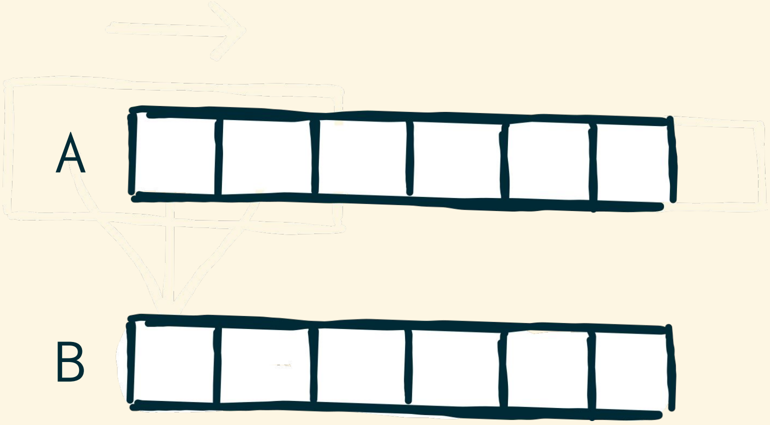
Can we express stencil computations in Lift?

Let's look at a simple stencil example...

WHAT ARE STENCIL COMPUTATIONS?

3-point-stencil.c

```
for (int i = 0; i < N ; i ++) {
    int sum = 0;
    for ( int j = -1; j <= 1; j ++) {
        int pos = i + j;
        pos = pos < 0 ? 0 : pos;
        pos = pos > N - 1 ? N - 1 : pos;
        sum += A[ pos ]; }
 B[ i ] = sum ; }
```



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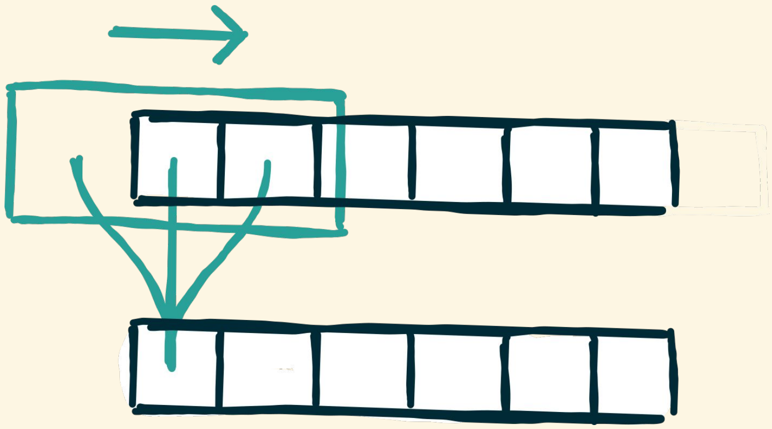

What are Stencil Computations?

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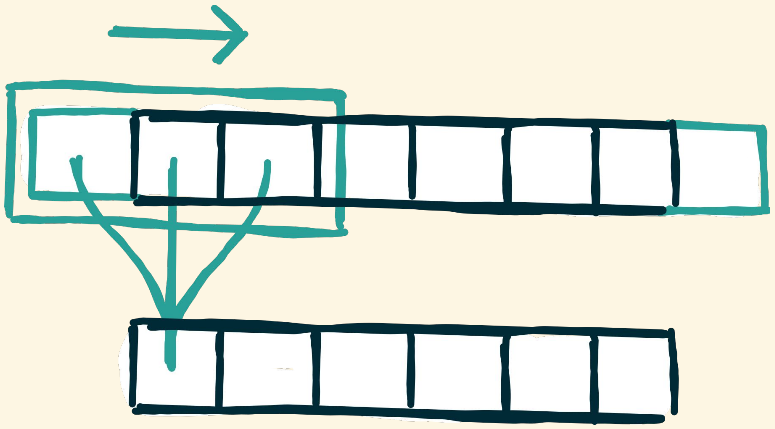
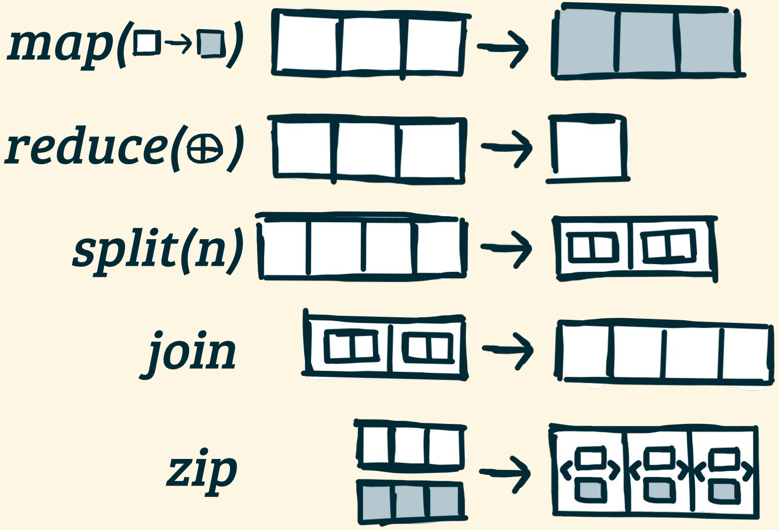
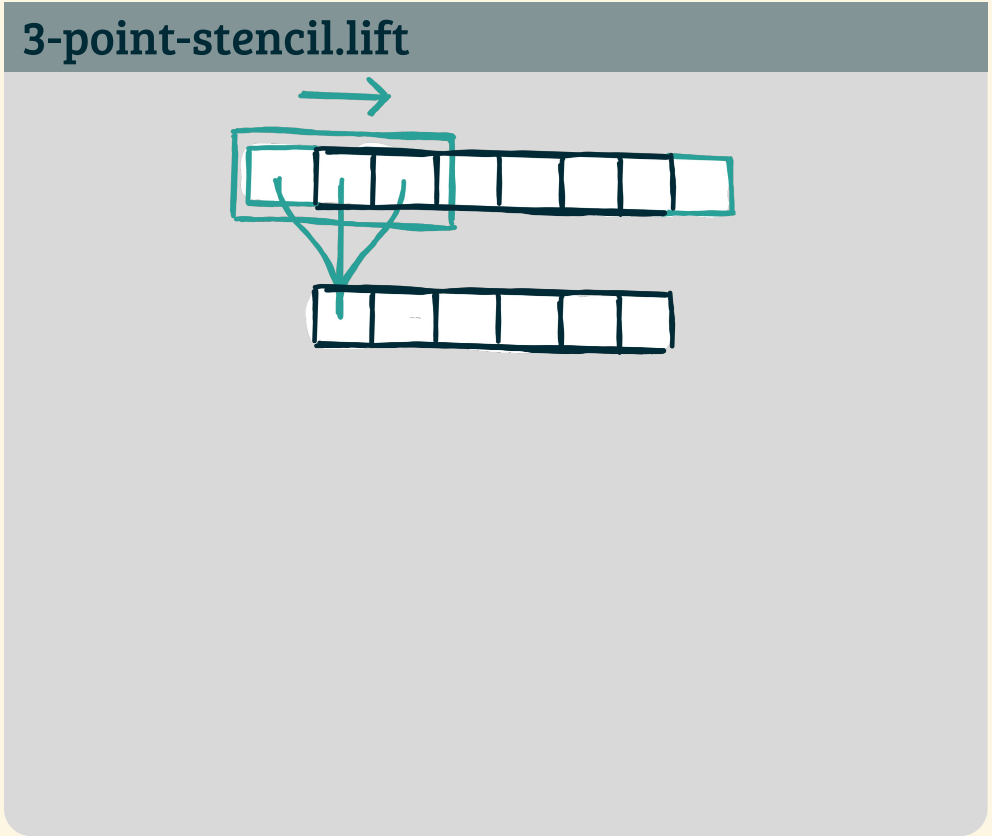
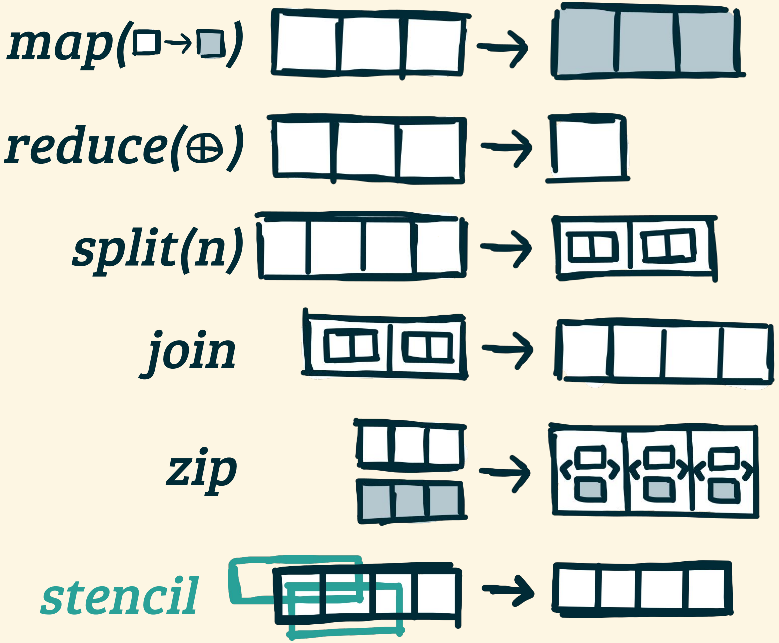


STENCIL COMPUTATIONS IN LIFT

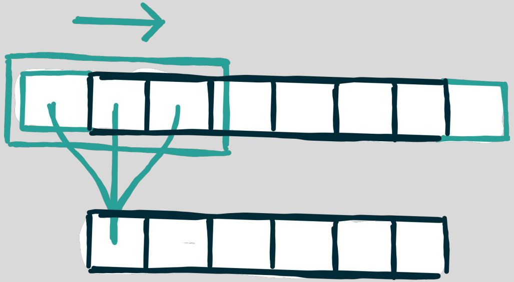




STENCIL COMPUTATIONS IN LIFT

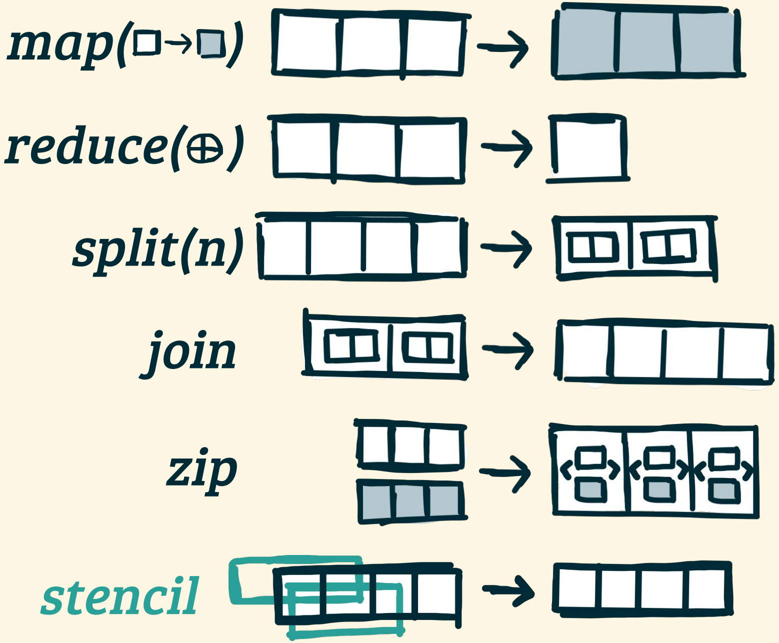


3-point-stencil.lift



Add specialized primitive: Job done?

STENCIL COMPUTATIONS IN LIFT



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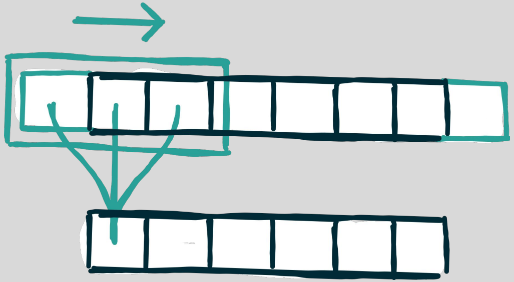

Add specialized primitive: Job done?

- **No Reuse**
  of existing primitives and optimizations
- **Domain-specific**
  rather than generic
- **Multidimensional?**
  is it composable?

DECOMPOSING STENCIL COMPUTATIONS

3-point-stencil.c

```c
for (int i = 0; i < N ; i ++) {
    int sum = 0;
    for ( int j = -1; j <= 1; j ++) {
        int pos = i + j;
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        sum += A[ pos ]; }
 B[ i ] = sum ; }
```
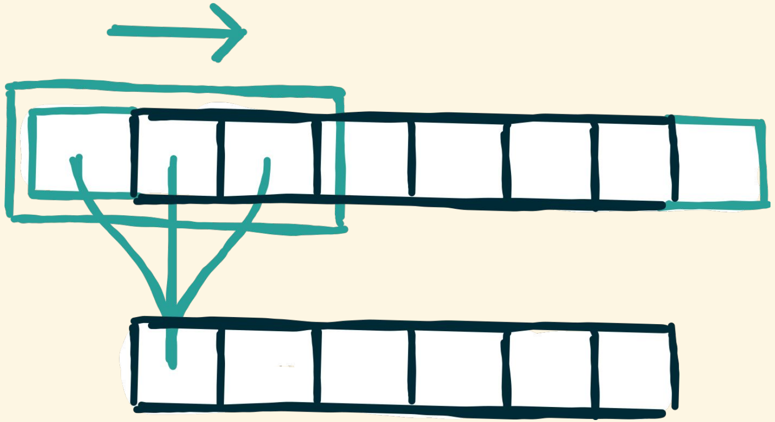
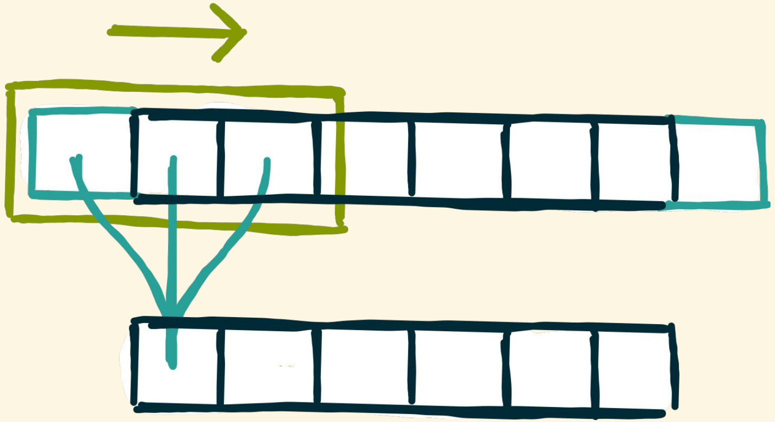



DECOMPOSING STENCIL COMPUTATIONS

3-point-stencil.c

```c
for (int i = 0; i < N ; i ++) {
    int sum = 0;
    for ( int j = -1; j <= 1; j ++) {    // ( a )
        int pos = i + j;
        pos = pos < 0 ? 0 : pos;
        pos = pos > N - 1 ? N - 1 : pos;
        sum += A[ pos ]; }
 B[ i ] = sum ; }
```



(a) access **neighborhoods** for every element

DECOMPOSING STENCIL COMPUTATIONS

3-point-stencil.c

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for (int i = 0; i < N ; i ++) {
    int sum = 0;
    for ( int j = -1; j <= 1; j ++) {   // ( a )
        int pos = i + j;
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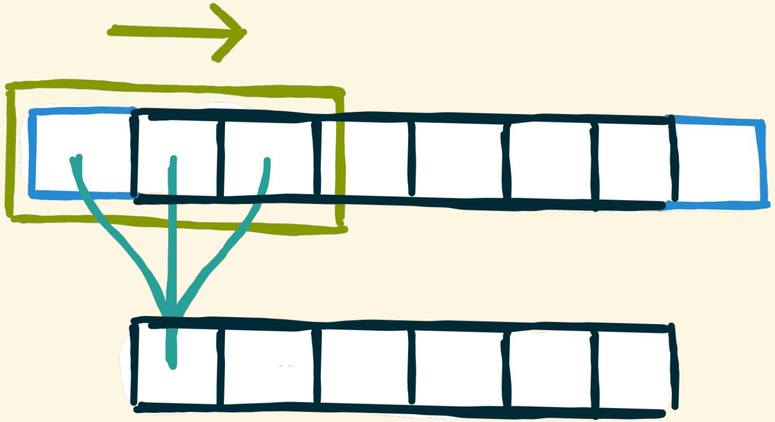
 B[ i ] = sum ; }
```



(a) access neighborhoods for every element

(b) specify boundary handling

DECOMPOSING STENCIL COMPUTATIONS

3-point-stencil.c

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for (int i = 0; i < N ; i ++) {
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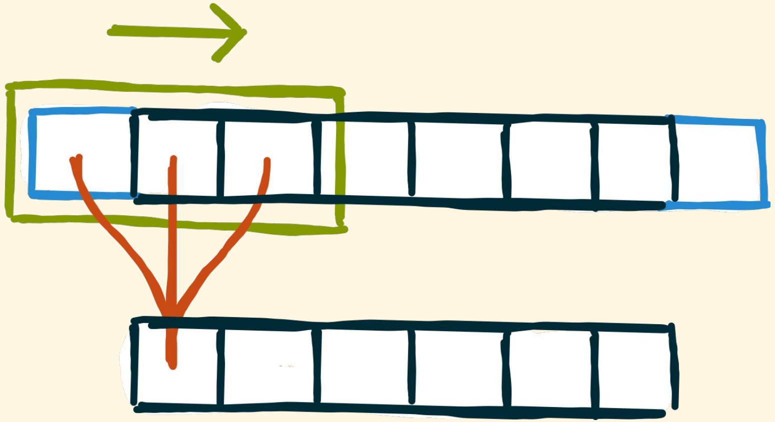
    for ( int j = -1; j <= 1; j ++) {   // ( a )
        int pos = i + j;
        pos = pos < 0 ? 0 : pos;        // ( b )
        pos = pos > N - 1 ? N - 1 : pos;
        sum += A[ pos ]; }              // ( c )
 B[ i ] = sum ; }
```



(a) access neighborhoods for every element
(b) specify boundary handling
(c) apply stencil function to neighborhoods

DECOMPOSING STENCIL COMPUTATIONS

3-point-stencil.c

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for (int i = 0; i < N ; i ++) {
    int sum = 0;
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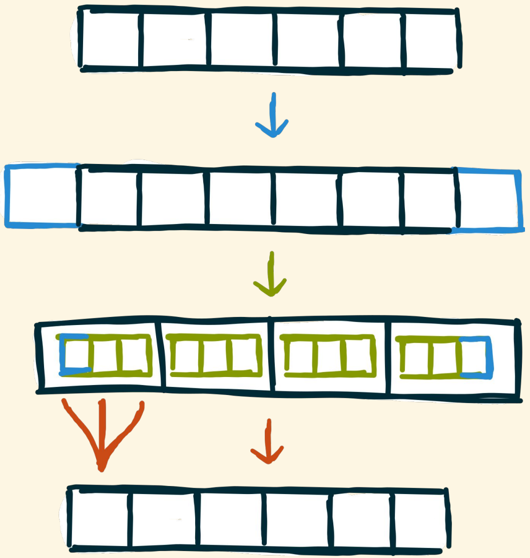
        int pos = i + j;
        pos = pos < 0 ? 0 : pos;         // ( b )
        pos = pos > N - 1 ? N - 1 : pos;
        sum += A[ pos ]; }               // ( c )
 B[ i ] = sum ; }
```



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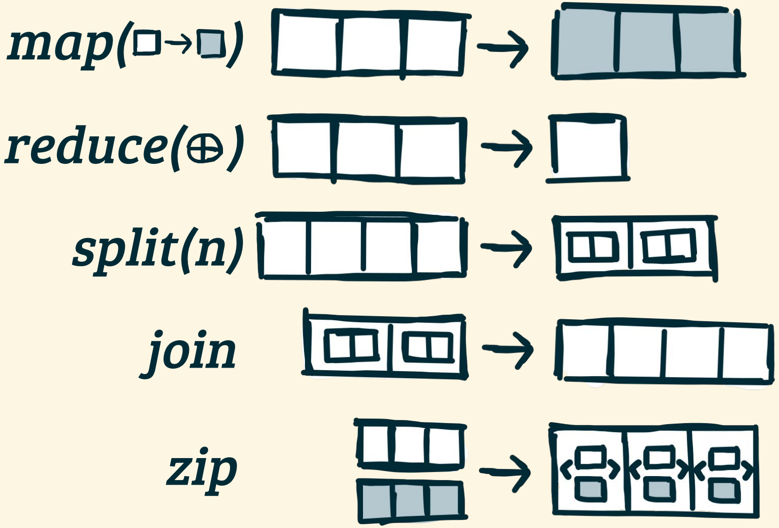
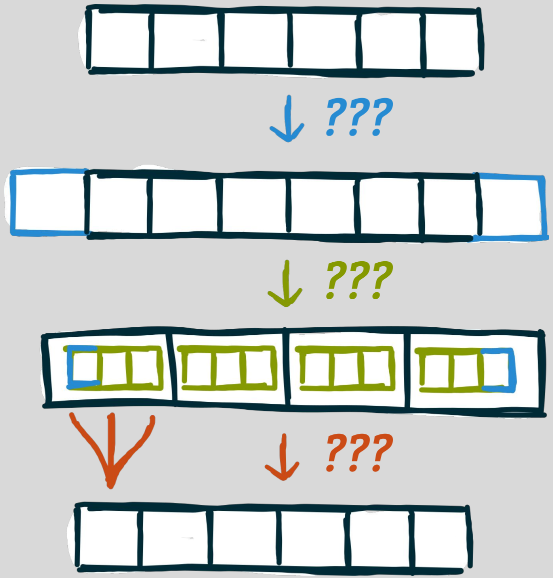
(b) specify boundary handling
(c) apply stencil function to neighborhoods

STENCIL COMPUTATIONS IN LIFT



3-point-stencil.lift



STENCIL COMPUTATIONS IN LIFT

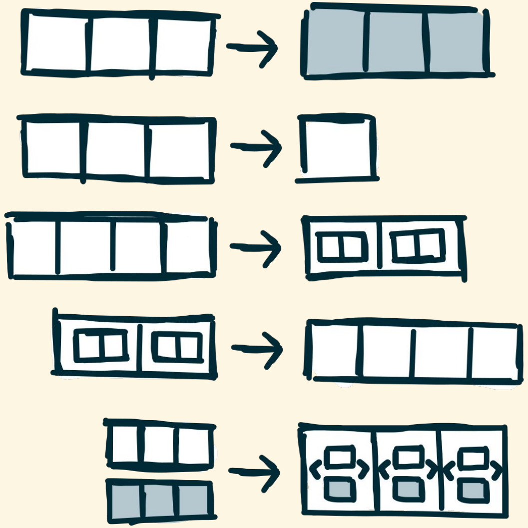
map(□→□)

reduce(⊕)

split(n)

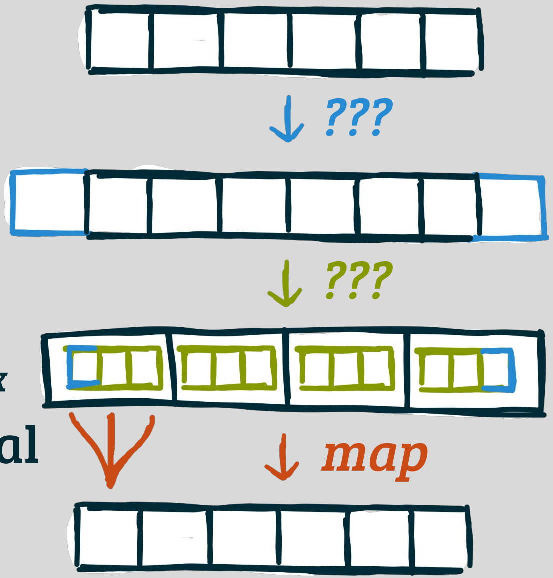
join

zip



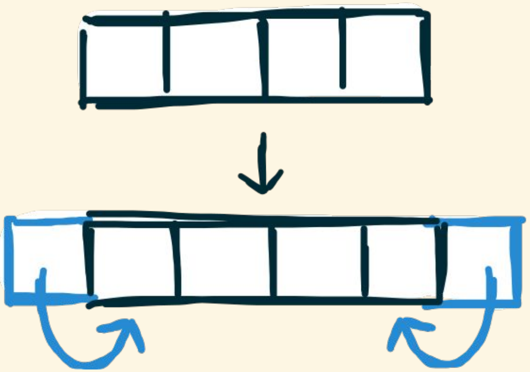
3-point-stencil.lift

- **Reuse *map***
  allows to reuse existing rewrite rules
- **Simplicity**
  one primitive per task
- **Multidimensional**
  easily composable

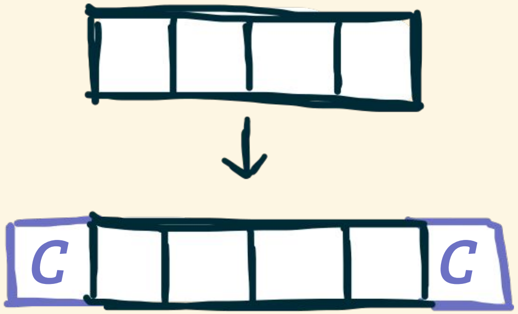


BOUNDARY HANDLING USING PAD

pad(reindexing)



pad(constant)



pad-reindexing.lift

```
clamp(i, n) = (i <  0) ? 0  :
              ((i >= n) ? n-1:i)

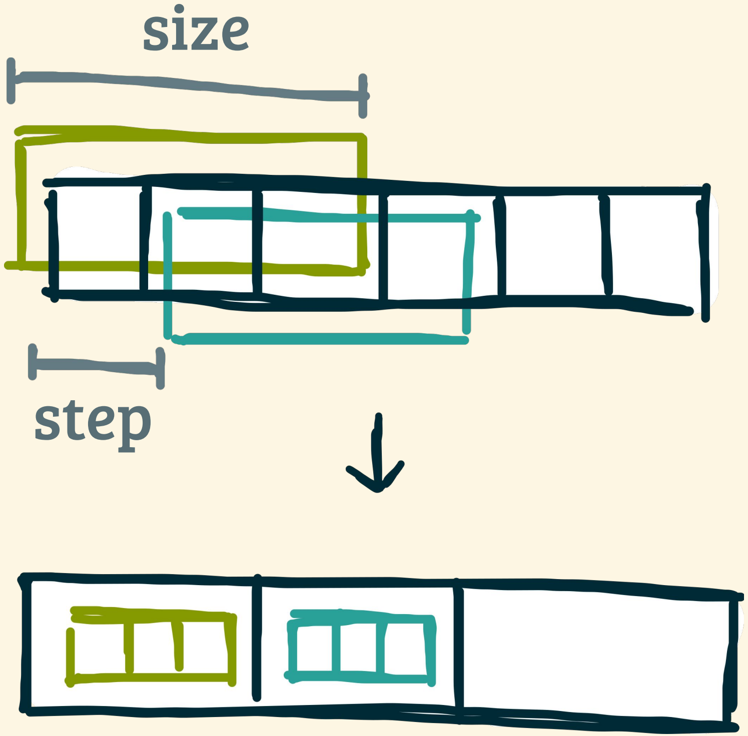
pad(1,1,clamp, [a,b,c,d]) =
    [a,a,b,c,d,d]
```

pad-constant.lift

```
constant(i, n) = C

pad(1,1,constant, [a,b,c,d]) =
    [C,a,b,c,d,C]
```

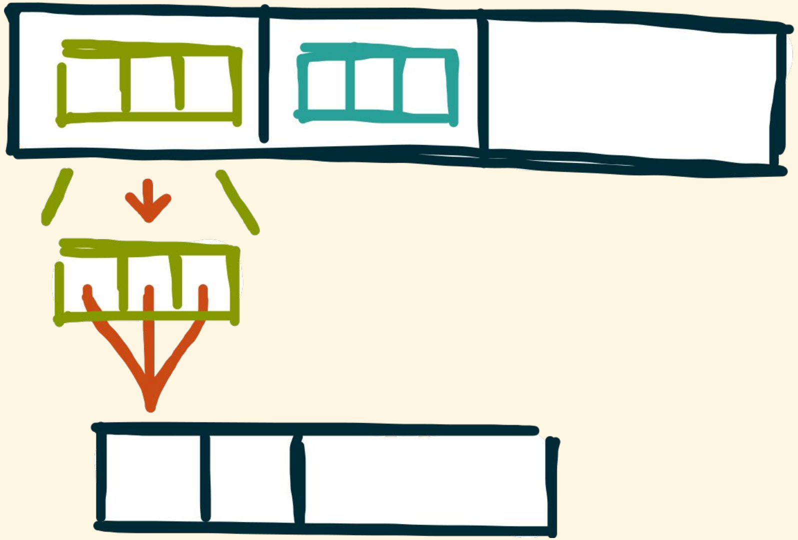
NEIGHBORHOOD CREATION USING SLIDE



slide-example.lift

```
slide(3,1,[a,b,c,d,e]) =
    [[a,b,c],[b,c,d],[c,d,e]]
```

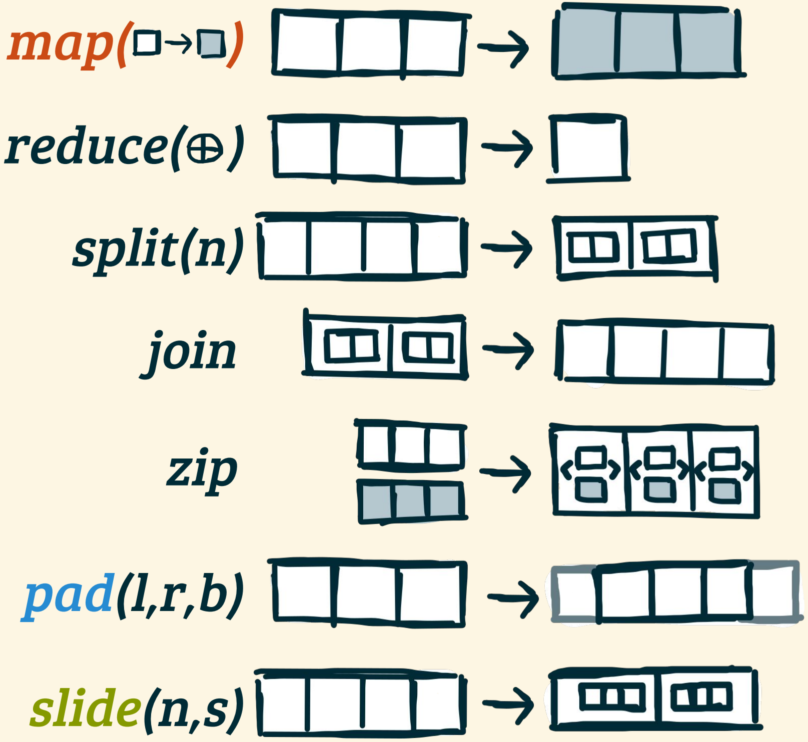
APPLYING STENCIL FUNCTION USING MAP



sum-neighborhoods.lift

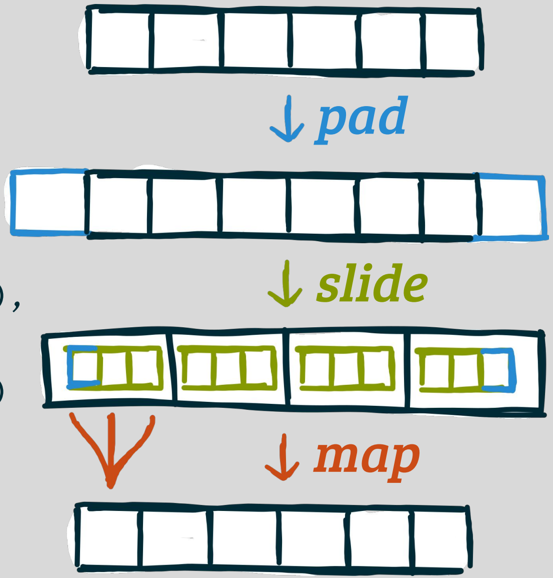
```
map(nbh =>
  reduce(add, 0.0f, nbh))
```

PUTTING IT TOGETHER



stencil1D.lift

```
def stencil1D =
 fun(A =>
  map(reduce(add, 0.0f),
   slide(3,1,
    pad(1,1,clamp,A))))
```



Multidimensional Stencil Computations

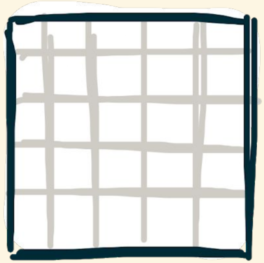
are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose

MULTIDIMENSIONAL STENCIL COMPUTATIONS

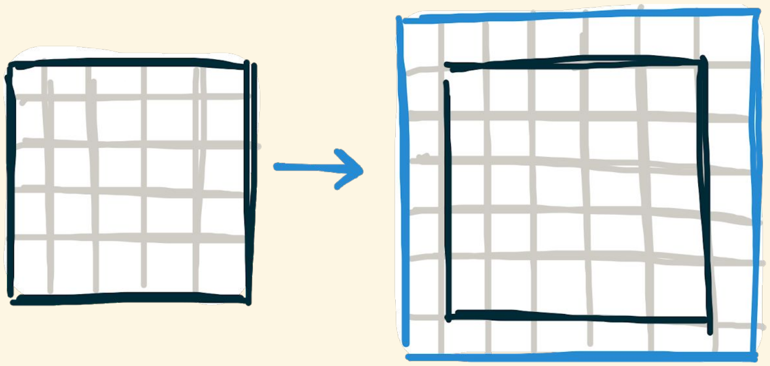
are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose



MULTIDIMENSIONAL STENCIL COMPUTATIONS

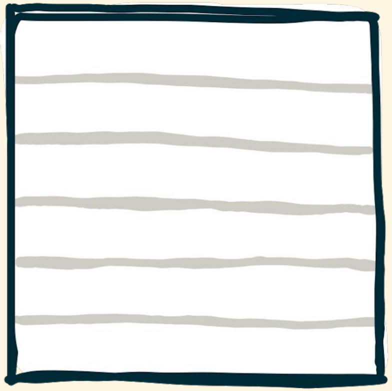
are expressed as compositions of intuitive, generic 1D primitives



$\mathbf{pad}_2(1, 1, \mathit{clamp}, \mathrm{input})$

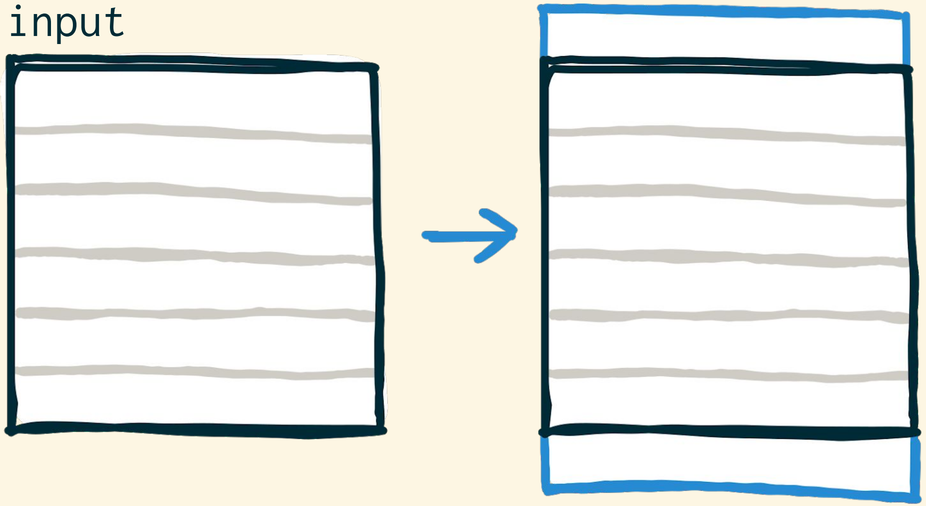
Multidimensional Boundary Handling using pad_2

input



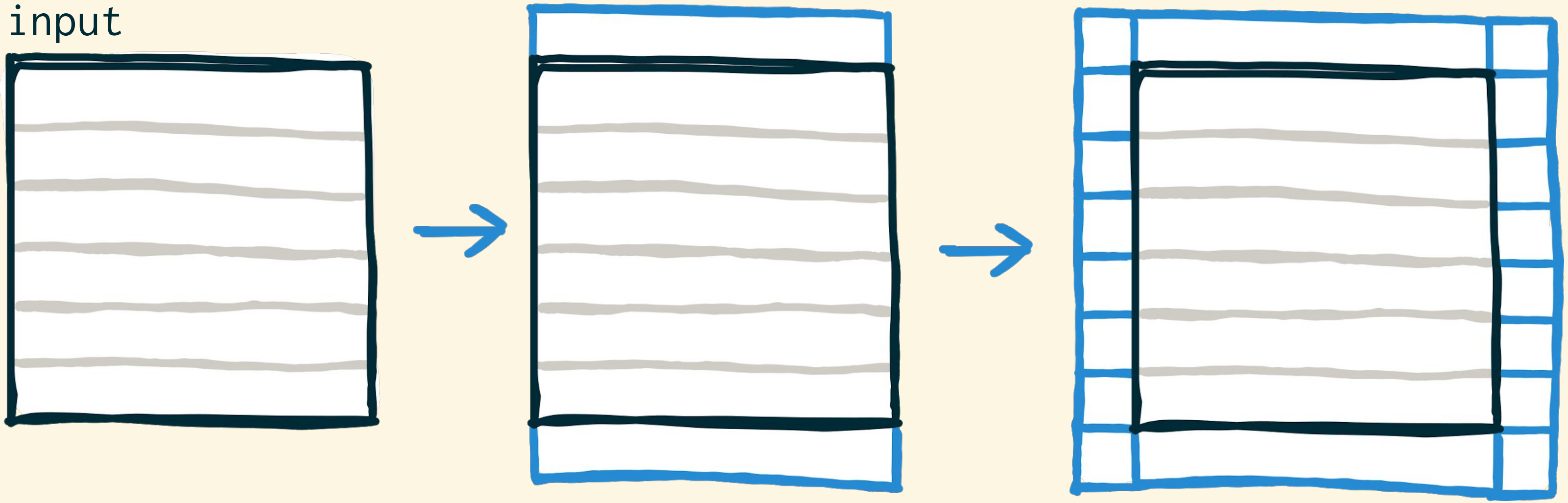
pad_2 =

MULTIDIMENSIONAL BOUNDARY HANDLING USING PAD_2



$pad_2 =$ pad(l,r,b,input)

MULTIDIMENSIONAL BOUNDARY HANDLING USING PAD_2

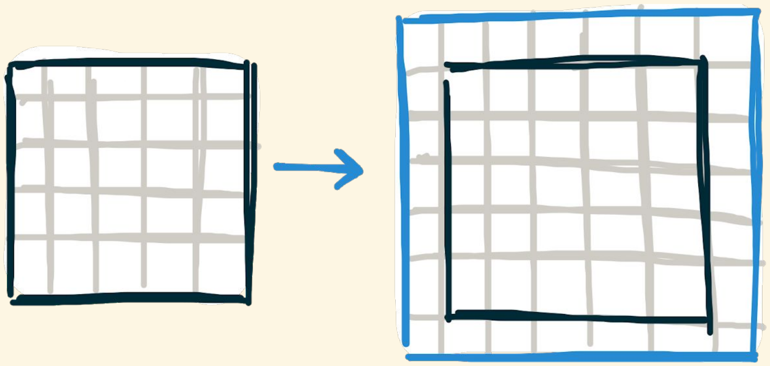


$$pad_2 = map(pad(l,r,b,pad(l,r,b,input)))$$

MULTIDIMENSIONAL STENCIL COMPUTATIONS

are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose

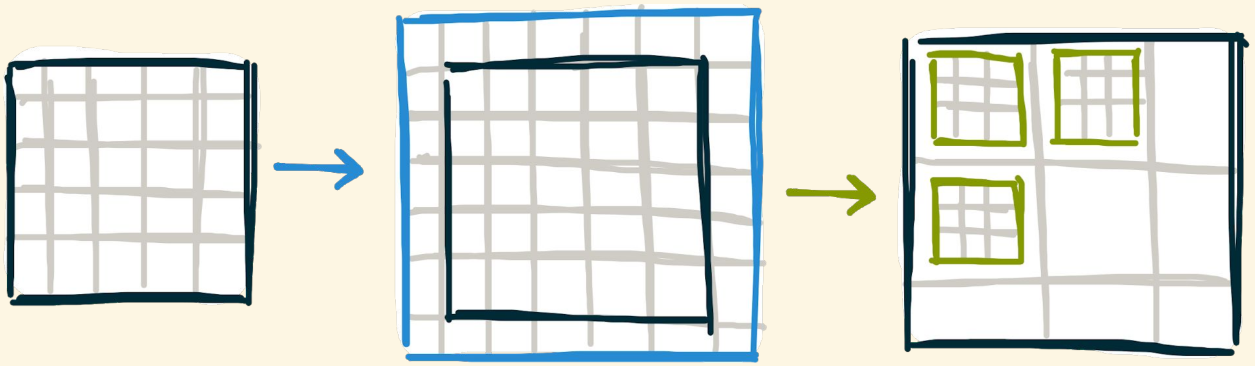


$\mathbf{pad}_2(1, 1, \mathit{clamp}, \mathtt{input})$

Multidimensional Stencil Computations

are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose

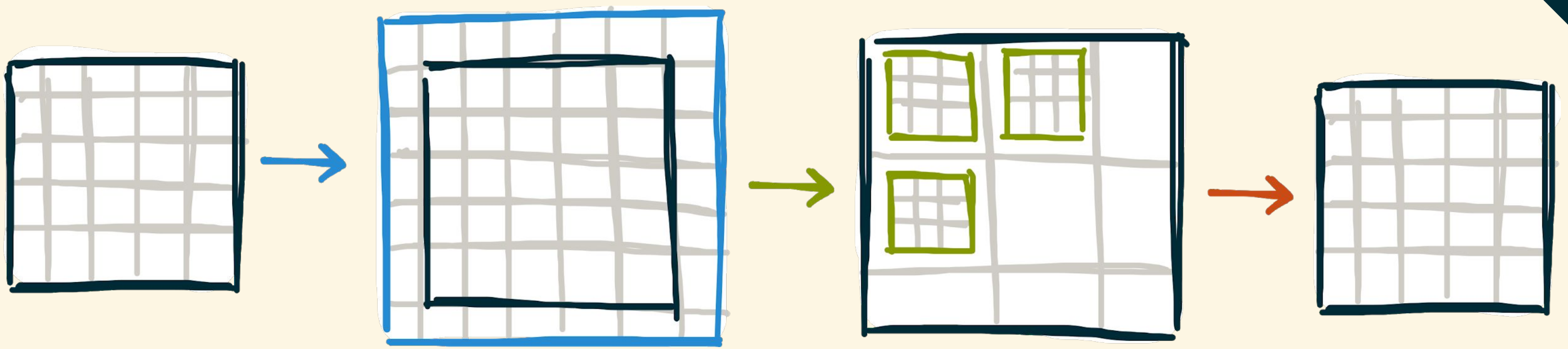


$\mathit{slide}_2(3,1,\ \mathit{pad}_2(1,1,\mathit{clamp},\mathrm{input}))$

MULTIDIMENSIONAL STENCIL COMPUTATIONS

are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose

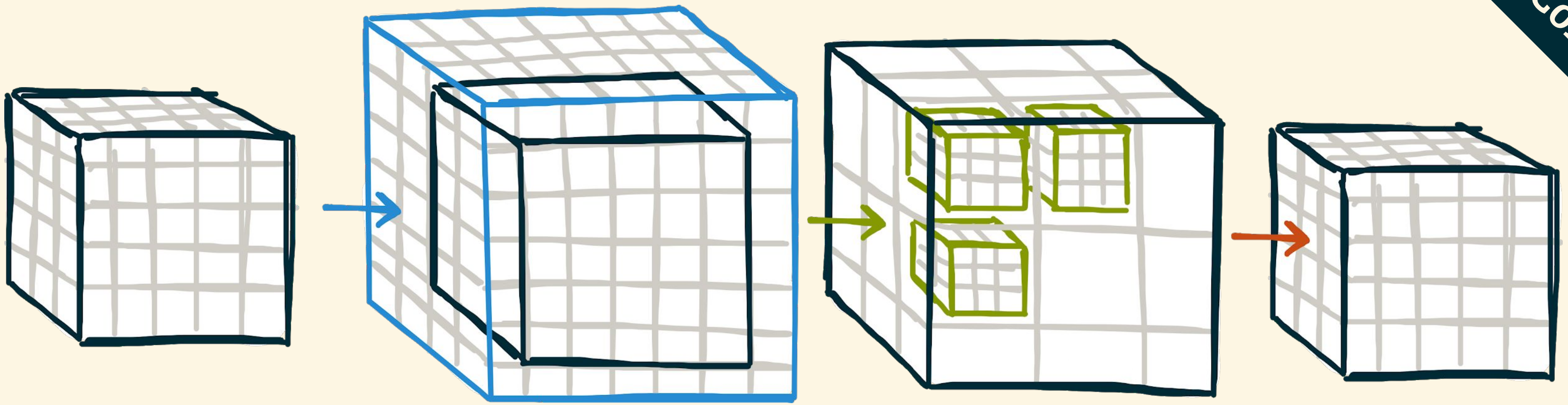


$map_2(sum,\ slide_2(3,1,\ pad_2(1,1,clamp,input)))$

MULTIDIMENSIONAL STENCIL COMPUTATIONS

are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose

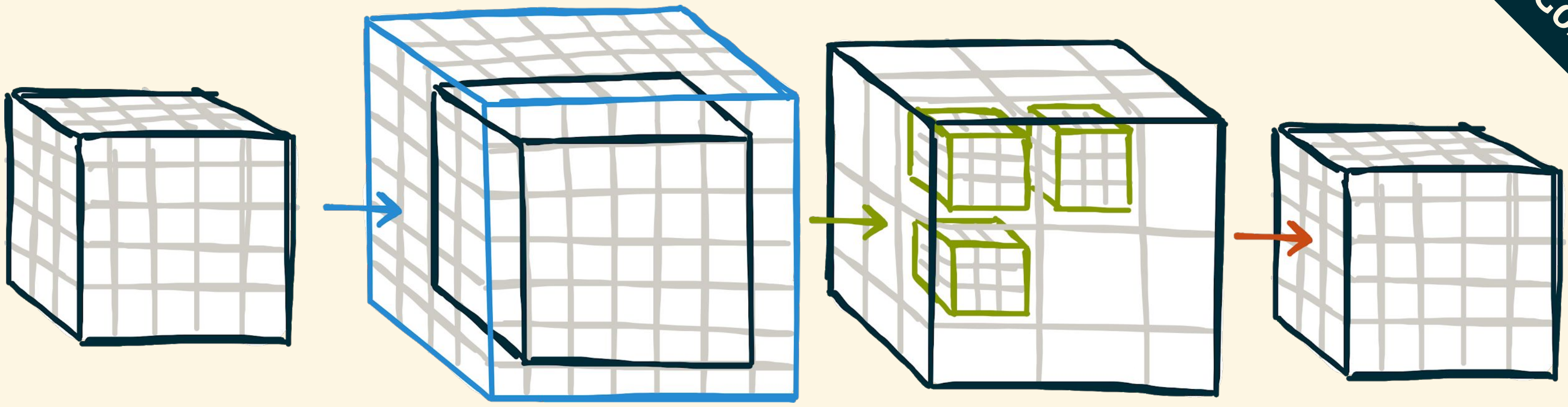


$map_3(sum,\ slide_3(3,1,\ pad_3(1,1,clamp,\mathrm{input})))$

MULTIDIMENSIONAL STENCIL COMPUTATIONS

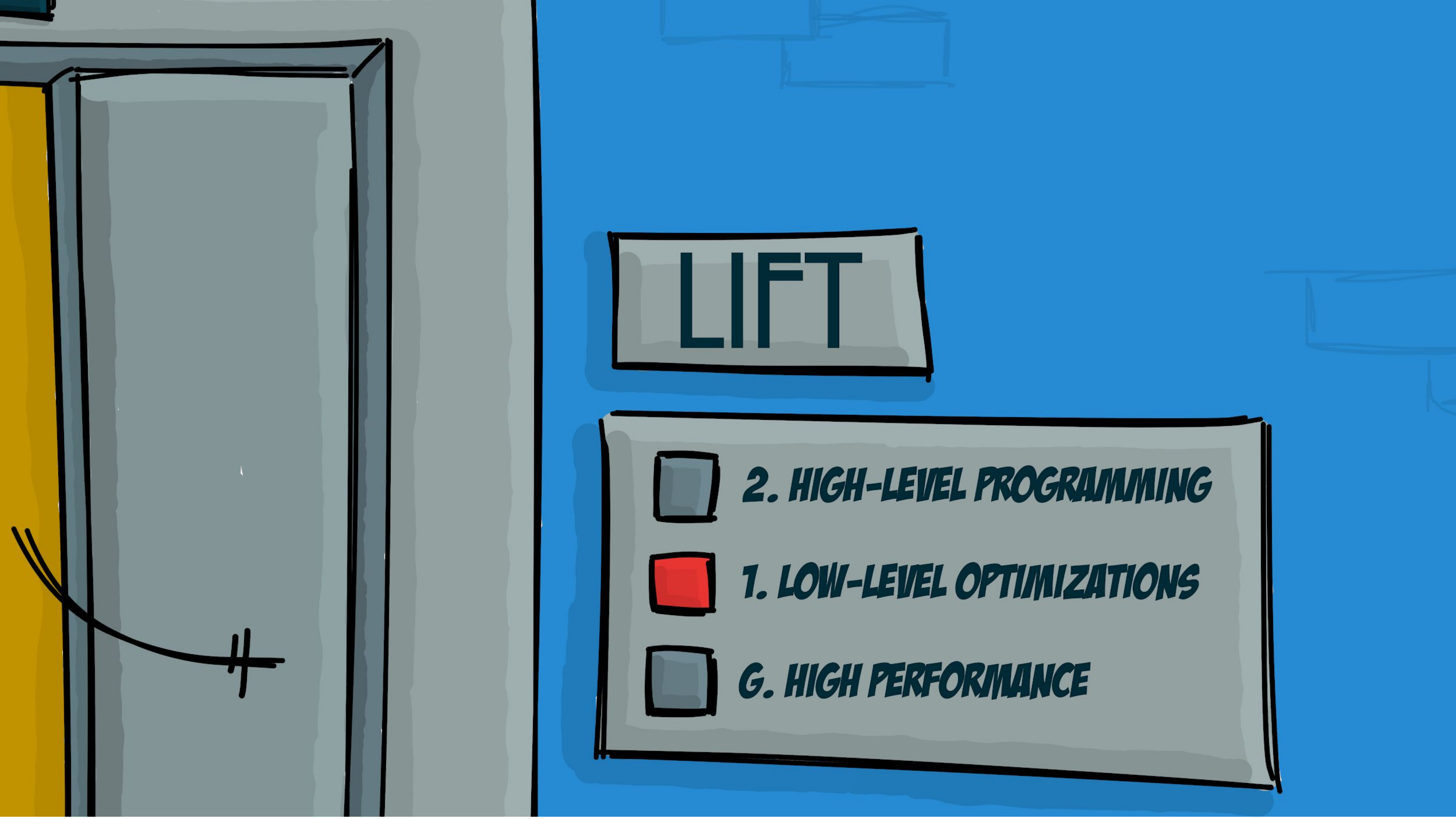
are expressed as compositions of intuitive, generic 1D primitives

Decompose to Re-Compose



$\mathit{map}_3(\mathit{sum},\ \mathit{slide}_3(3,1,\ \mathit{pad}_3(1,1,\mathit{clamp},\mathrm{input})))$

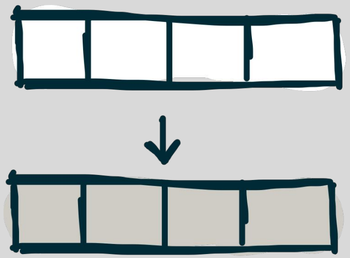
Advantages: ✔ Compact Language ✔ Reuse Rewrites ✔ Simple Compilation


LIFT
2. HIGH-LEVEL PROGRAMMING
1. LOW-LEVEL OPTIMIZATIONS
G. HIGH PERFORMANCE

REUSING EXISTING REWRITE RULES

Divide & Conquer

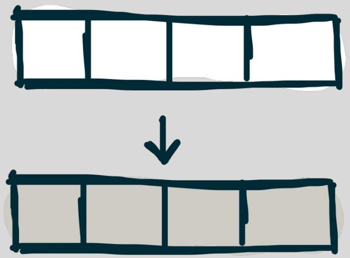
`map(f, A)`

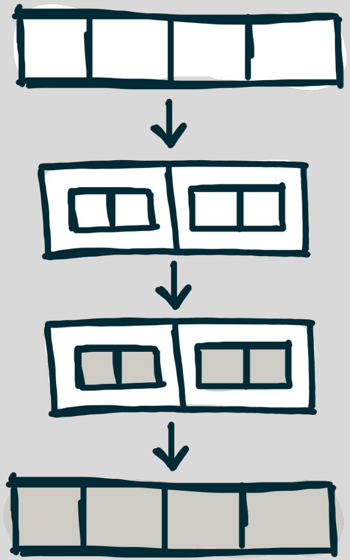


REUSING EXISTING REWRITE RULES

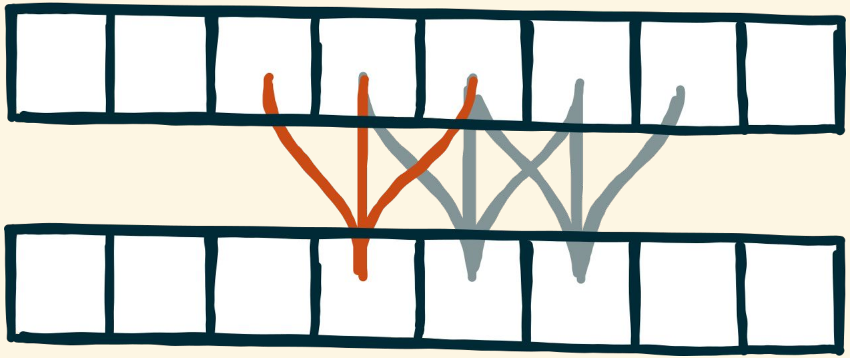
Divide & Conquer

map(f, A) $\mapsto$ *join*(*map*(*map*(f), *split*(n, A)))





OPTIMIZATION: OVERLAPPED TILING



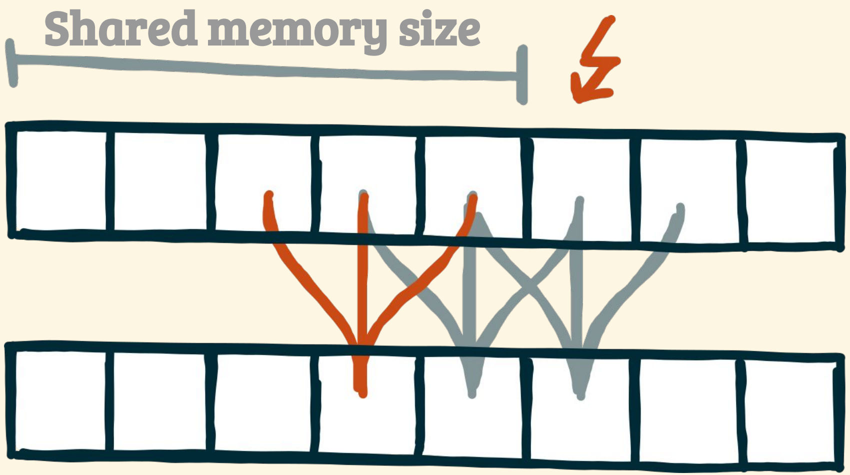
- **Exploit Locality**

  Close neighborhoods share elements that can be grouped in tiles

- **Shared Memory**

  Fast memory can be used to cache tiles

OPTIMIZATION: OVERLAPPED TILING



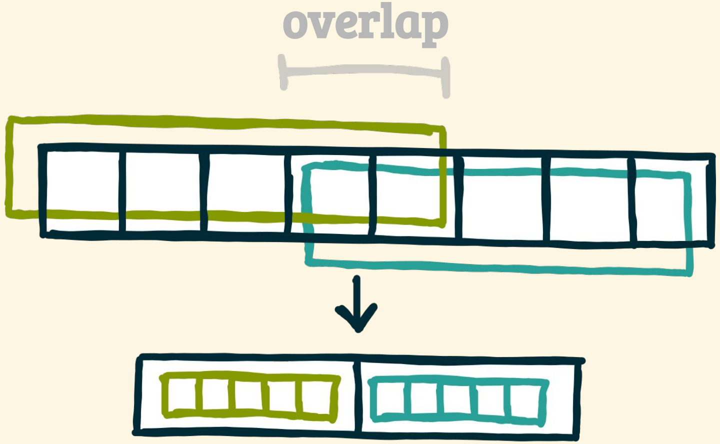
Exploit Locality

Close neighborhoods share elements that can be grouped in tiles

Shared Memory

Fast memory can be used to cache tiles

Optimization: Overlapped Tiling



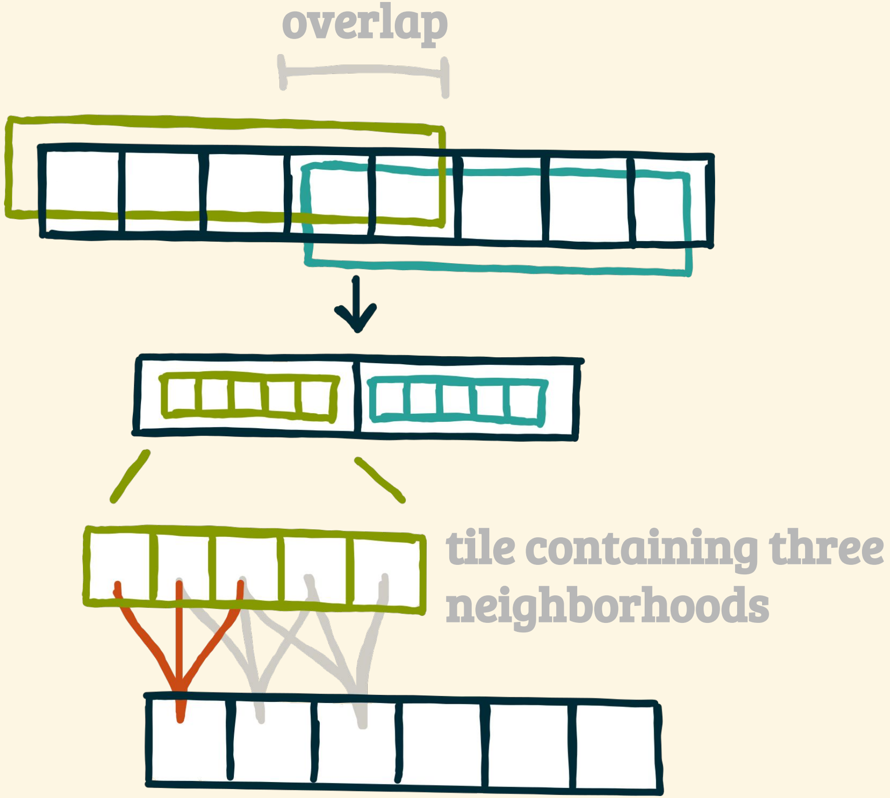
Exploit Locality

Close neighborhoods share elements that can be grouped in tiles

Shared Memory

Fast memory can be used to cache tiles

OPTIMIZATION: OVERLAPPED TILING



Exploit Locality

Close neighborhoods share elements that can be grouped in tiles

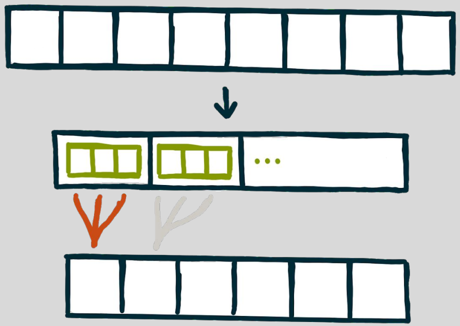
Shared Memory

Fast memory can be used to cache tiles

Overlapped Tiling as a Rewrite Rule

overlapped tiling rule

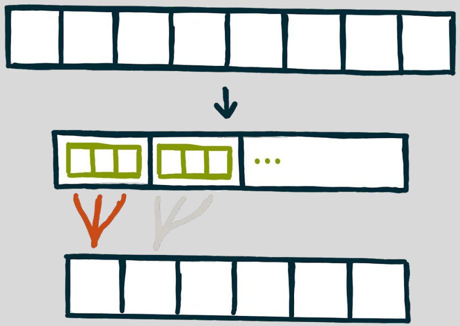
```
map(f, slide(3,1,input))
```



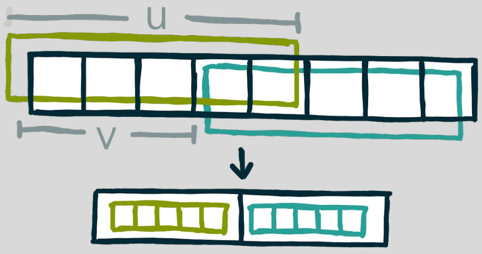
OVERLAPPED TILING AS A REWRITE RULE

overlapped tiling rule

map(f, *slide*(3,1,input)) ↦



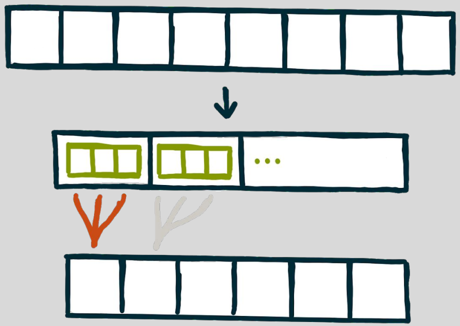
slide(u,v,input)

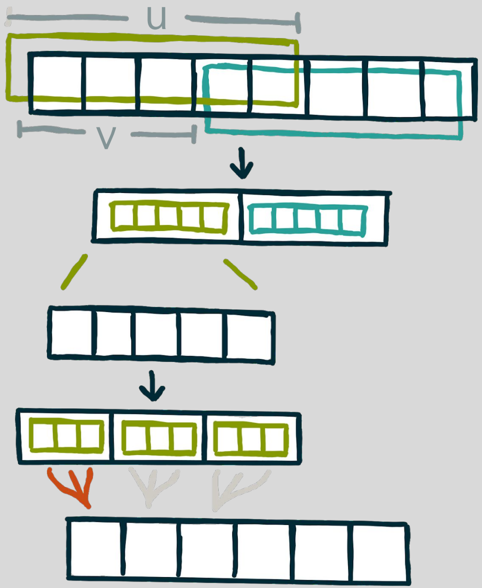


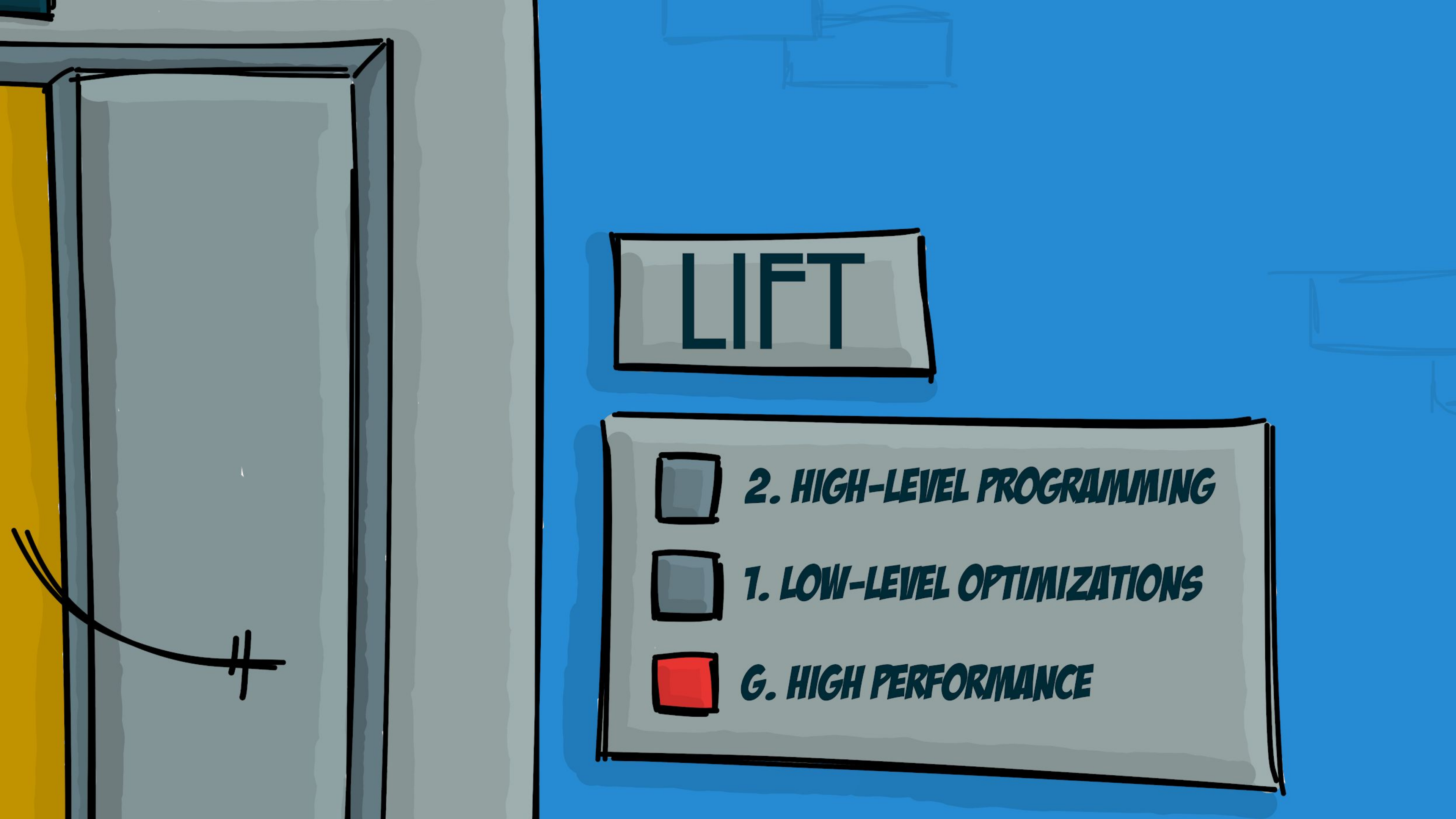
Overlapped Tiling as a Rewrite Rule

overlapped tiling rule

```
map(f, slide(3,1,input))  ↦  join(map(tile ⇒
                                 map(f, slide(3,1,tile)),
                                   slide(u,v,input)))
```



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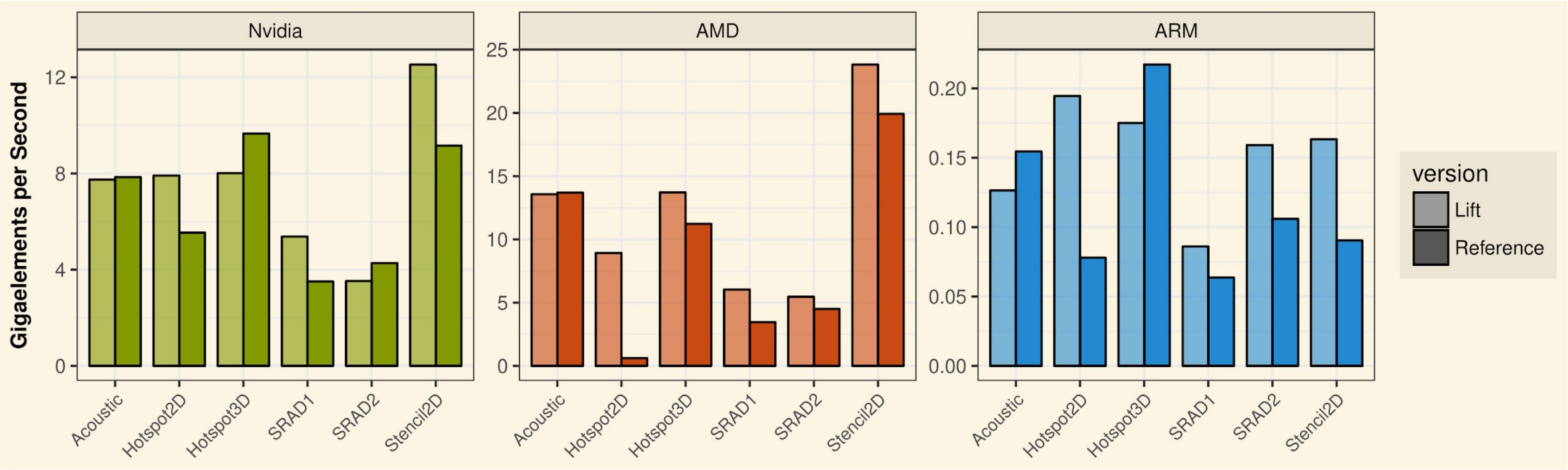
Comparison with Hand-Optimized Codes

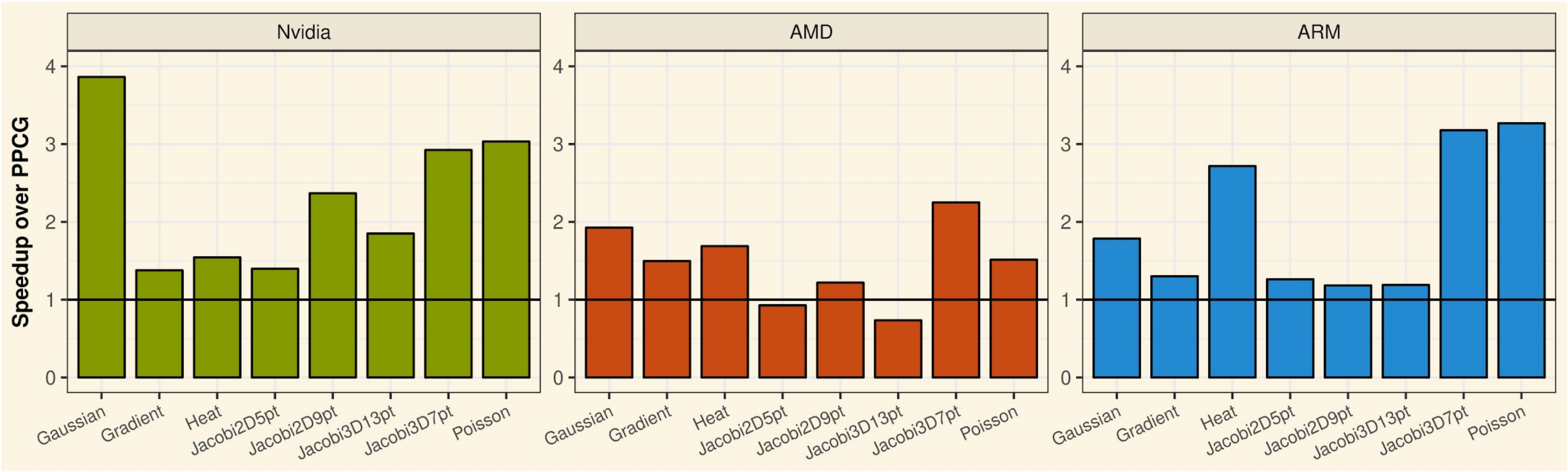
higher is better



Lift achieves the same performance as hand optimized code

COMPARISON WITH POLYHEDRAL COMPILATION

higher is better



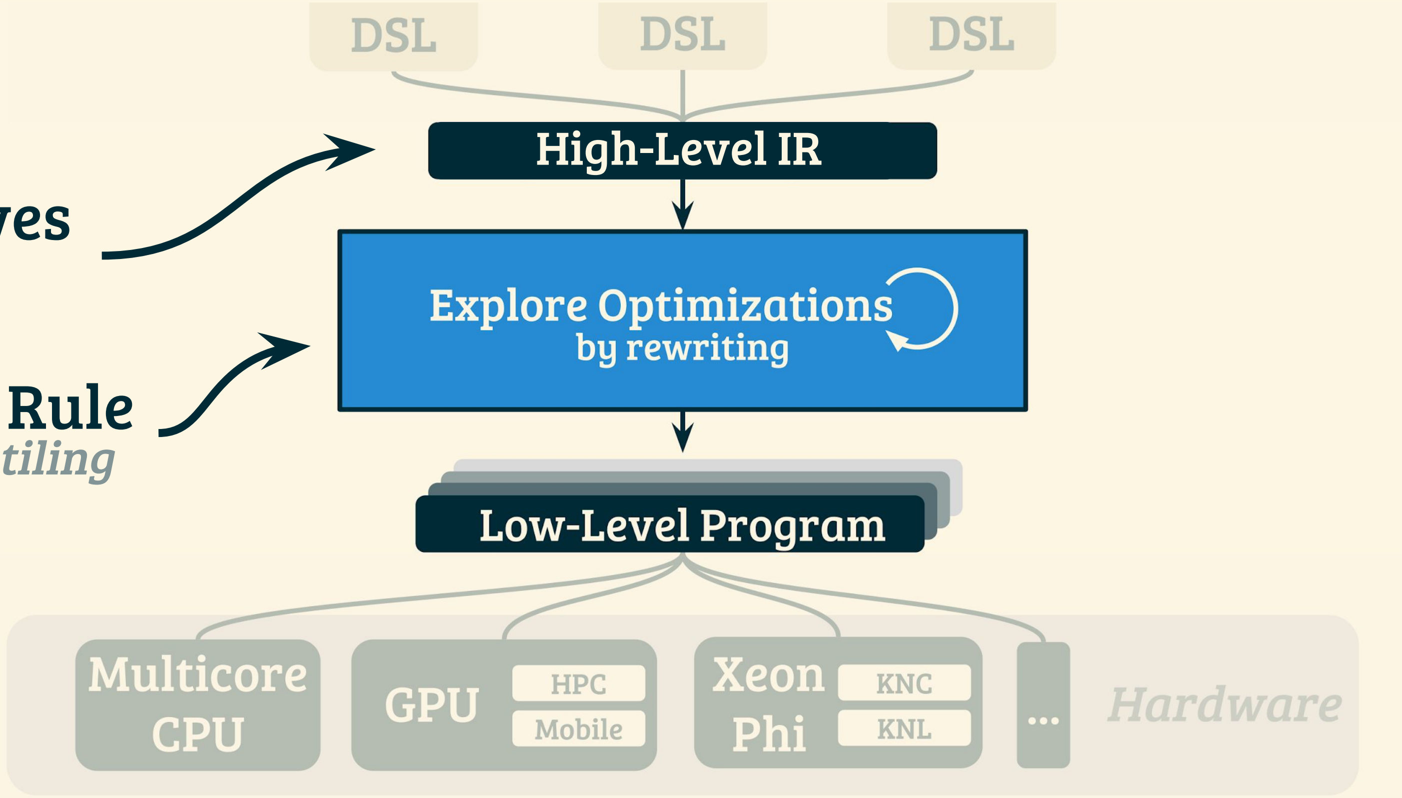
Lift outperforms state-of-the-art optimizing compilers

STENCIL COMPUTATIONS IN LIFT

We added:

2 Primitives
pad, *slide*

1 Rewrite Rule
overlapped tiling



LIFT IS OPEN SOURCE!

more info at:

lift-project.org

Paper CGO Artifact Source Code

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